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2006 AAS/AIAA Space Flight Mechanics Meeting

22-26 Janurary 2006
Tampa, Florida EUA

DECENTRALIZED, LOW-COMMUNICATION STATE ESTIMATION AND OPTIMAL GUIDANCE OF FORMATION FLYING SPACECRAFT

Dan Dumitriu, Sónia Marques, Pedro Lima, Bogdan Udrea

ESA (European Space Agency) project RFQ/3-10624/03/NL/LvH/bj
"Formation Estimation Methodologies for Distributed Spacecraft"

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Plan of the presentation

- **Introduction**
- **Relative Dynamics for Eccentric Orbits**
- **Guidance and Control**
  - **Optimal Trajectory Planning problem**
  - **Closed-loop GC algorithm**
- **Navigation**
  - **Measurements and Relative state vector**
  - **Full State Decentralized Problem**
  - **Covariance Intersection**
  - **Full algorithm**
- **Simulation results**
- **Conclusions**

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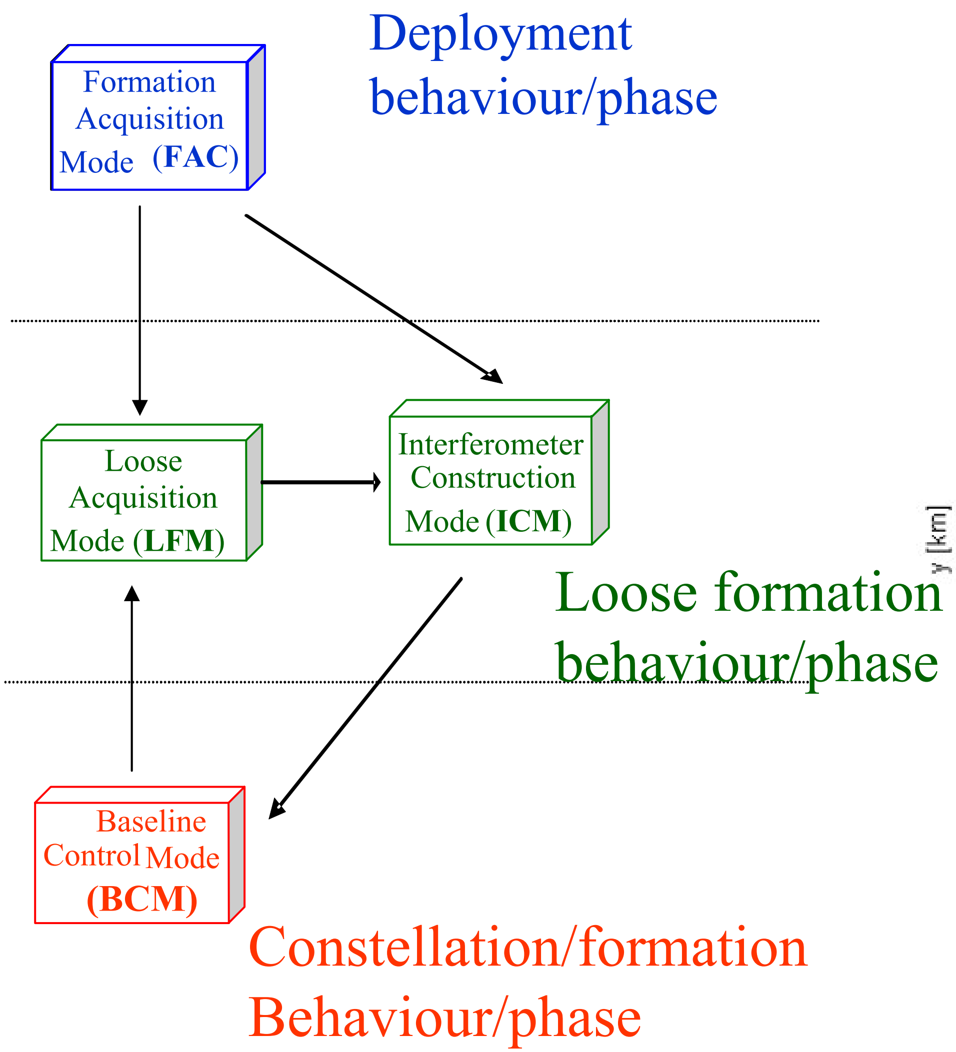
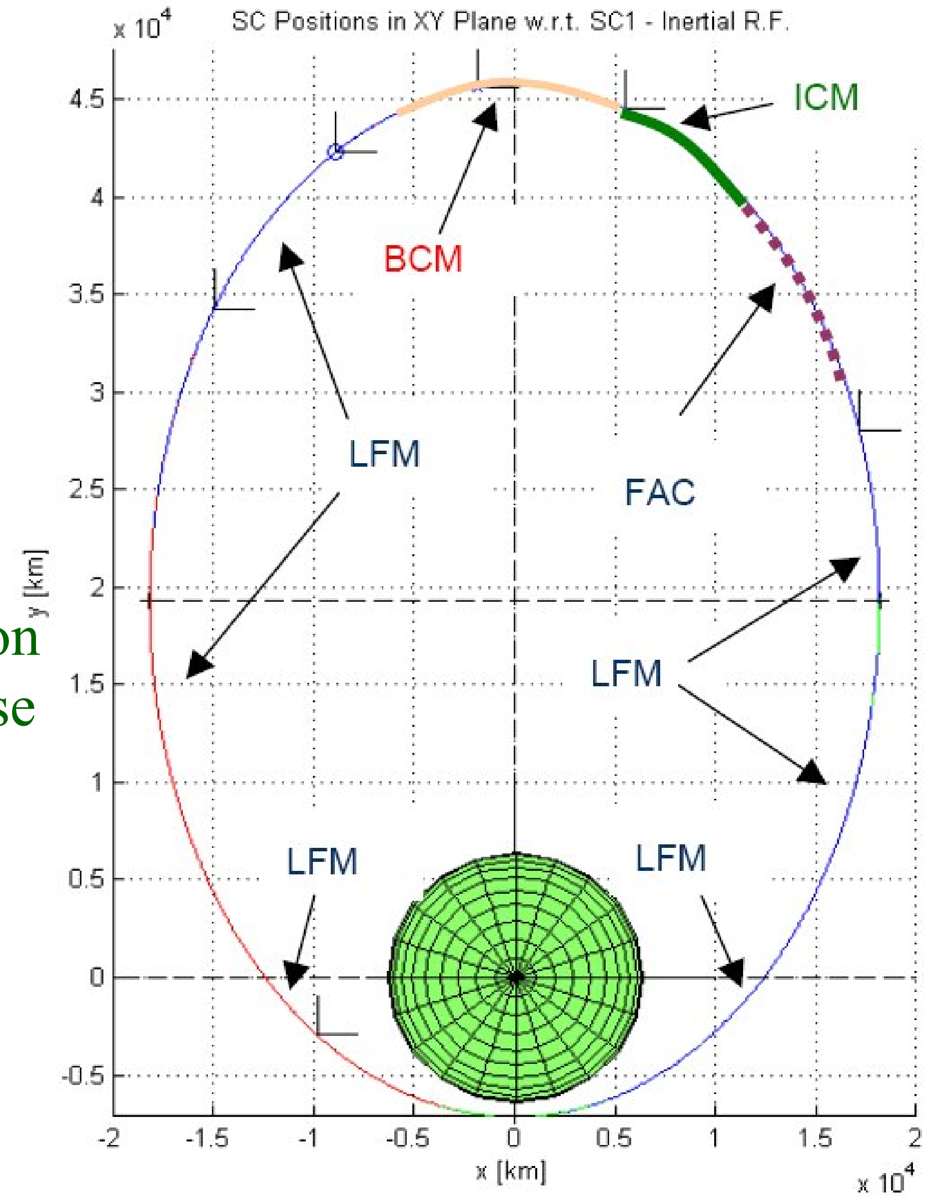
Introduction → *GTO Mission*

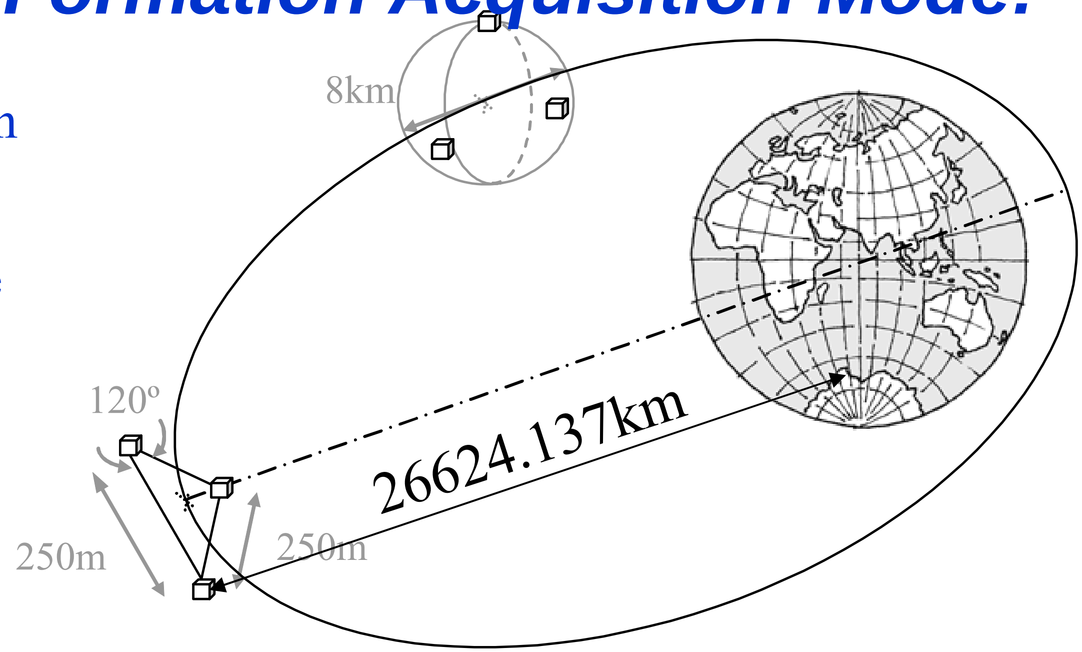




Mission goal during Formation Acquisition Mode:

Formation Flying demonstration mission in a GTO orbit:

- Science experiments in apogee
- 3 spacecraft Formation Flying
- 1orbit period:12hours



→ from an initial *random* disposition (at θ_1=beginning of FAM)

within a sphere of 8km in diameter centered in the dispenser, the relative velocities being null (with a random error of ±0.1m/s added)

→ to the desired final disposition at θ_2 (end of FAM),

i.e., an isosceles triangle formation with the equal edges of 250m and with a 120º angle between them.

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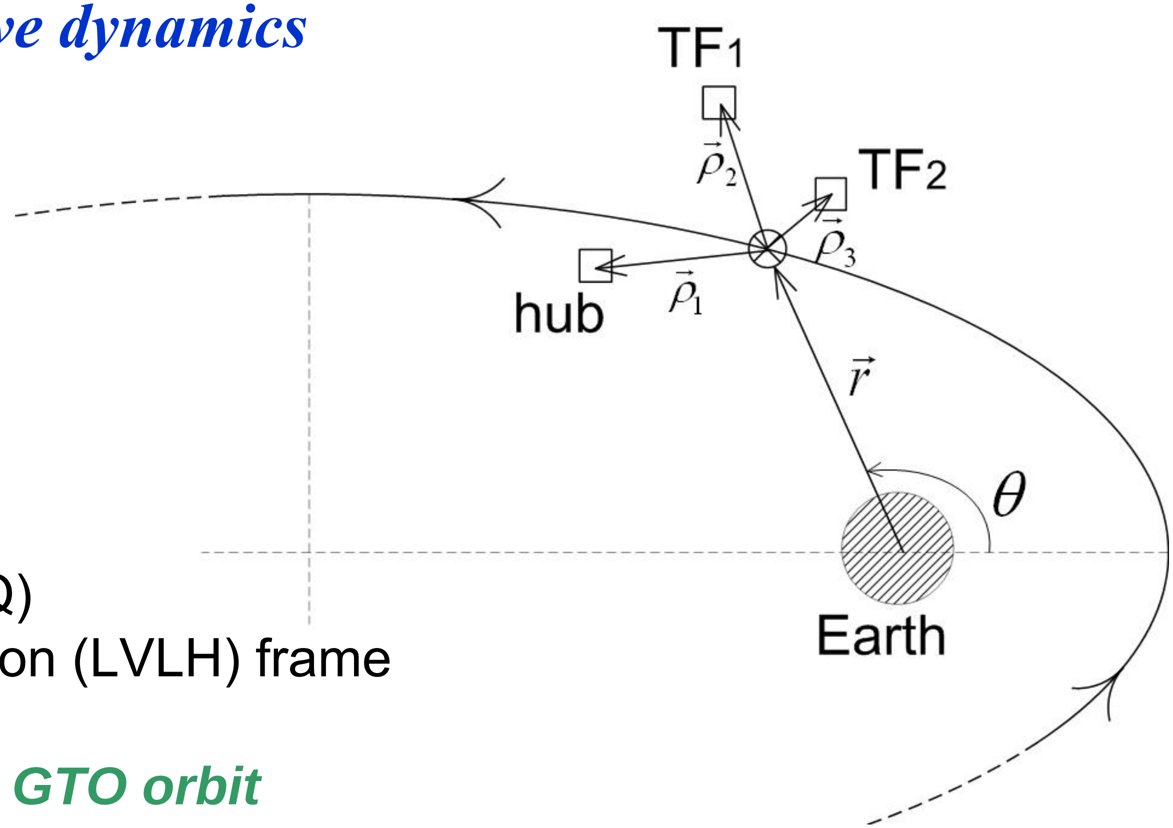
Illustration of the relative dynamics parameters that we use

$$\vec{\rho}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \quad for\ i = 1,2,3$$



2 reference frames

- Inertial Planet Frame (IPQ)
- Local Vertical Local Horizon (LVLH) frame

Orbital parameters of the GTO orbit

- semi-major axis a, eccentricity e, RAAN Ω,
- inclination i, argument of perigee ω, true anomaly θ

$$t - t_p = \frac{1}{n}\left[2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right) - \frac{e\sqrt{1-e^2}\sin\theta}{1+e\cos\theta}\right]$$

period

$T = 12\text{h } 34\text{s}$

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Linearized θ-varying relative dynamics equations (in LVLH)

- In-plane motion of *i* th spacecraft ($i=1,2,3$)

$$\frac{d}{d\theta}\begin{bmatrix} x_i \\ x_i' \\ z_i \\ z_i' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{e\cos\theta}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} & \frac{-2e\sin\theta}{1+e\cos\theta} & 2 \\ 0 & 0 & 0 & 1 \\ \frac{2e\sin\theta}{1+e\cos\theta} & -2 & \frac{3+e\cos\theta}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} \end{bmatrix}\begin{bmatrix} x_i \\ x_i' \\ z_i \\ z_i' \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_{i,x} + \sum w_{i,x} \\ u_{i,z} + \sum w_{i,z} \end{bmatrix}$$

- Out-of-plane motion

$$\frac{d}{d\theta}\begin{bmatrix} y_i \\ y_i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} \end{bmatrix}\begin{bmatrix} y_i \\ y_i' \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(u_{i,y} + \sum w_{i,y}\right)$$

- **Perturbations in GTO orbit**: J_2 effect, third-body (Sun, Moon) gravitational differential perturbations, micrometeoroids, atmospheric drag, solar radiation pressure

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FAM Optimal Trajectory Planning problem

- Non-linear θ-varying dynamics equations (State equations)

$$\frac{d[\mathbf{X}(\theta)]}{d\theta} = \mathbf{A}(\theta)\mathbf{X}(\theta) + \mathbf{B}(\theta)\left[\mathbf{U}(\theta) + \mathbf{W}(\theta)\right]$$

- Initial and final conditions (two-boundary conditions)

- Limitations concerning the control inputs

$$u_{\min} \leq |U_j| \leq u_{\max}, \quad with \;\; u_{\min} = 0.1\mu\text{N}, \quad u_{\max} = 17\text{mN}$$

- The cost function to be minimized, taking into account the propellant consumption

$$J = \int_{\theta_1}^{\theta_2} L\left(\mathbf{X}(\theta), \mathbf{U}(\theta), \theta\right) d\theta = \int_{\theta_1}^{\theta_2} \sum_{j=1}^{9} U_j^2 d\theta$$

⇒ Guidance and Control are handled simultaneously. In fact, the current approach is purely Guidance, and we re-plan regularly!

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Pontryagin's maximum principle (PMP) to solve the optimal trajectory planning problem

Hamiltonian: $$H(\mathbf{X},\mathbf{U},\theta) = L(\mathbf{X},\mathbf{U},\theta) + \sum_{k=1}^{18} \lambda_k f_k(\mathbf{X},\mathbf{U},\theta)$$

+ co-state equations introduced

The control inputs which satisfy, for all $\theta_1 \leq \theta \leq \theta_2$, the stationarity conditions, are the optimal control inputs, the corresponding trajectory being optimal as well !

Stationarity conditions

$$U_j^{opt} = -\frac{1}{2}\frac{(1-e^2)^3}{(1+e\cos(\theta))^4 n^2}\lambda_{2j}\ , \quad for \quad j = 1,\ldots,9$$

So, the optimal control inputs U_j^{opt} are linked to the adjoint variables λ_i of the PMP formulation by these linear relations!

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Closed-loop GC algorithm to solve the Optimal Trajectory Planning problem

➢ The closed-loop GC algorithm = algebraic version of the iterative shooting method. The algorithm is simple and reliable ⇒ very few convergence troubles

● needs little computing-time, much less than 1s (Pentium4 3.0GHz)

➢ The control inputs limitations and collision avoidance are considered a posteriori. Perturbations are not considered.

➢ To take the unmodeled perturbations + the state estimation errors into account, the algorithm is recomputed periodically, at regularly spaced time instants, and the planned trajectory is updated !

Example: For a 6h FAM, during the first 5h we execute the algorithm every 500s. In the last hour, we execute it every 100s.

GNC for spacecraft formation → Closed-loop GC algorithm

- The differential state equations (without perturbations) are:

$$\left.\frac{d\mathbf{X}_i}{d\theta}\right|_{\theta_k} = \mathbf{A}_i(\theta_k)\mathbf{X}_i(\theta_k) + \mathbf{B}_i^{\Lambda}(\theta_k)\mathbf{\Lambda}_i(\theta_k) \qquad \left.\frac{d\mathbf{X}_i}{d\theta}\right|_{\theta_k} = \frac{\mathbf{X}_i(k+1)-\mathbf{X}_i(k)}{\delta\theta}$$

$$\Rightarrow \mathbf{X}_i(k+1) = \overline{\mathbf{A}}_i(k)\mathbf{X}_i(k) + \overline{\mathbf{B}}_i(k)\mathbf{\Lambda}_i(k)$$

➢ *for the adjoint variables vector:* $\mathbf{\Lambda}_i(k+1) = \overline{\mathbf{C}}_i(k)\mathbf{\Lambda}_i(k)$

$\mathbf{X}_i(k+1)$ and $\mathbf{\Lambda}_i(k+1)$ expressed directly as function of $\mathbf{X}_i(0)$ and $\mathbf{\Lambda}_i(0)$:

$$\mathbf{X}_i(k+1) = \mathbf{P}_i(k)\mathbf{X}_i(0) + \mathbf{Q}_i(k)\mathbf{\Lambda}_i(0)$$

$$\mathbf{\Lambda}_i(k+1) = \mathbf{N}_i(k)\mathbf{\Lambda}_i(0)$$

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Recurrent sequence (⇔ propagating dynamics)

1. ☞ $\mathbf{P}_i(0)=\overline{\mathbf{A}}_i(0),\quad \mathbf{Q}_i(0)=\overline{\mathbf{B}}_i(0),\quad \mathbf{N}_i(0)=\overline{\mathbf{C}}_i(0)$

2. ☞ FOR *k*=1 TO *n*-1

$$\mathbf{P}_i(k)=\overline{\mathbf{A}}_i(k)\mathbf{P}_i(k-1)$$

$$\mathbf{Q}_i(k)=\overline{\mathbf{A}}_i(k)\mathbf{Q}_i(k-1)+\overline{\mathbf{B}}_i(k)\mathbf{N}_i(k-1)$$

$$\mathbf{N}_i(k)=\overline{\mathbf{C}}_i(k)\mathbf{N}_i(k-1)$$

➢ trajectory planned between $\theta_1=\theta_{k=0}$ and $\theta_2=\theta_{k=n}$, with: $n=\dfrac{\theta_2-\theta_1}{\delta\theta}$

$$\Rightarrow \quad \mathbf{Q}_i(n-1)\mathbf{\Lambda}_i(0)=\mathbf{X}_i(\theta_2)-\mathbf{P}_i(n-1)\hat{\mathbf{X}}_i(\theta_1)$$

⇔ ***Algebraic system of 6 (4+2, decoupled) linear equations (unknowns $\mathbf{\Lambda}_i(0)$), can be solved analytically.***

➢ *We know $\mathbf{\Lambda}_i(0)$ ⇒ we know all $\mathbf{\Lambda}_i(\theta)$ ⇒ we know all $\mathbf{U}_i(\theta)$ (optimal control inputs), by the stationarity conditions*

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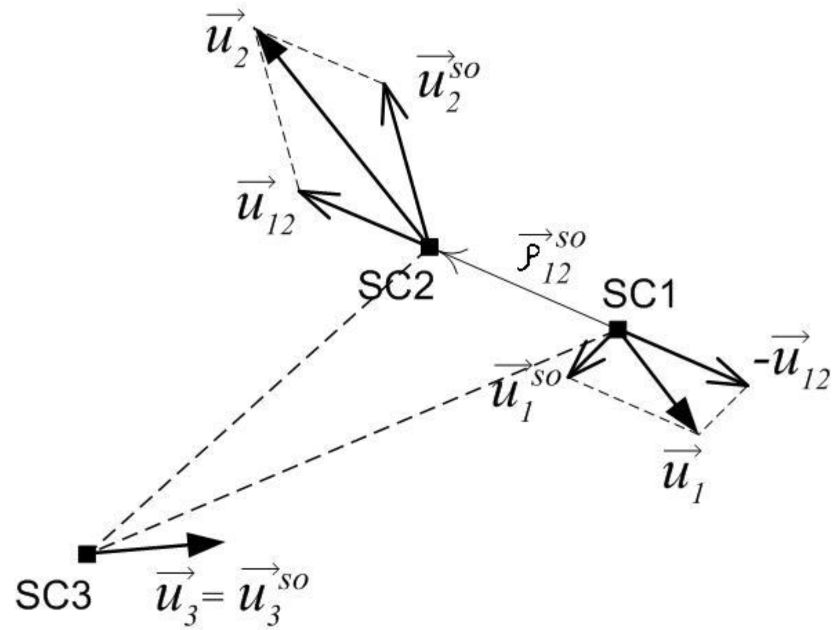
A posteriori consideration of collision avoidance

- EXAMPLE:

IF $\left|\vec{\rho}_{12}^{\,so}\right| < \rho_{\min} = 40m$

THEN $\vec{u}_1 = \vec{u}_1^{\,so} - \vec{u}_{12}, \quad \vec{u}_2 = \vec{u}_2^{\,so} + \vec{u}_{12},$

$\vec{u}_3 = \vec{u}_3^{\,so}$



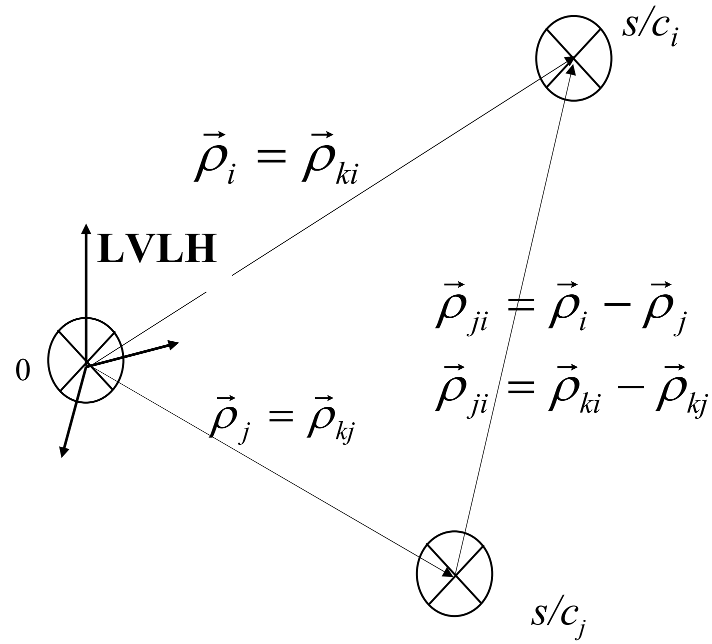
Control inputs limitations

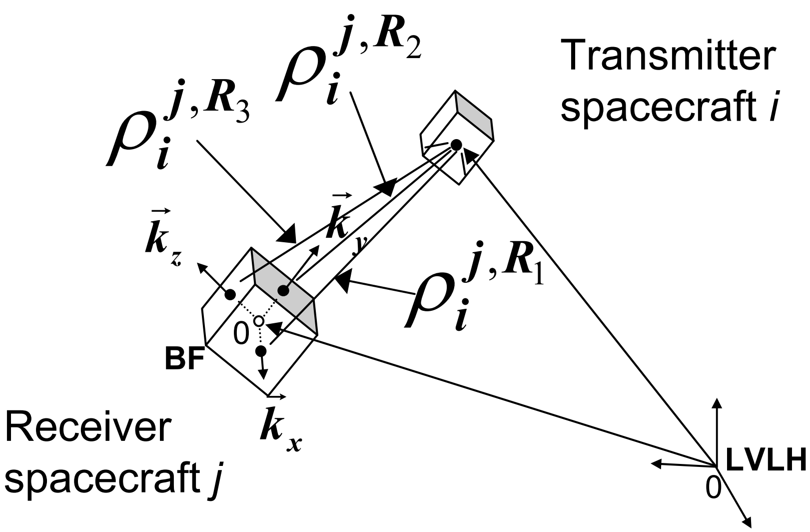
IF $|u_{1,x}| > u_{\max}$ THEN $|u_{1,x}| = u_{\max}$

- **State vector: Relative variables**

$$\chi = \left[(\vec{\rho}_{12})^T \quad (\vec{\rho}_{32})^T \quad (\vec{\rho}_{31})^T \quad (\vec{\rho}\,'_{12})^T \quad (\vec{\rho}\,'_{32})^T \quad (\vec{\rho}\,'_{31})^T \right]^T$$

$$= \left[\begin{bmatrix} x_{12} \\ y_{12} \\ z_{12} \end{bmatrix}^T \begin{bmatrix} x_{32} \\ y_{32} \\ z_{32} \end{bmatrix}^T \begin{bmatrix} x_{31} \\ y_{31} \\ z_{31} \end{bmatrix}^T \begin{bmatrix} x'_{12} \\ y'_{12} \\ z'_{12} \end{bmatrix}^T \begin{bmatrix} x'_{32} \\ y'_{32} \\ z'_{32} \end{bmatrix}^T \begin{bmatrix} x'_{31} \\ y'_{31} \\ z'_{31} \end{bmatrix}^T \right]^T$$





$$\left\| \vec{\rho}_i - \vec{\rho}_j \right\|^{R_1} = \sqrt{(x_{ij} - ap)^2 + y_{ij}^2 + z_{ij}^2}$$

$$\left\| \vec{\rho}_i - \vec{\rho}_j \right\|^{R_2} = \sqrt{x_{ij}^2 + (y_{ij} - ap)^2 + z_{ij}^2}$$

$$\left\| \vec{\rho}_i - \vec{\rho}_j \right\|^{R_3} = \sqrt{x_{ij}^2 + y_{ij}^2 + (z_{ij} - ap)^2}$$

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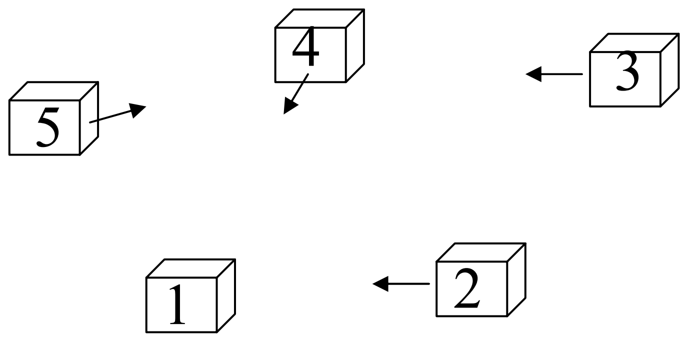


$$\chi_1 = \left[(\vec{\rho}_{12})^T \quad (\vec{\rho}_{32})^T \quad (\vec{\rho}_{31})^T \quad (\vec{\rho}_{14})^T \quad (\vec{\rho}_{15})^T \quad (\vec{\rho}_{54})^T \right]^T$$

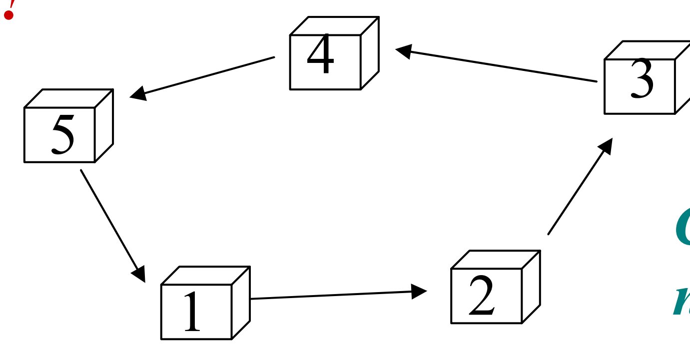
$$\chi_2 = \left[(\vec{\rho}_{12})^T \quad (\vec{\rho}_{32})^T \quad (\vec{\rho}_{31})^T \quad (\vec{\rho}_{14})^T \quad (\vec{\rho}_{15})^T \quad (\vec{\rho}_{54})^T \right]^T$$

Measurement network

How to estimate the *full state* at each spacecraft in a *decentralized* manner?

$$\begin{cases} z_1 = \boldsymbol{x} + \boldsymbol{v}_5 \\ z_2 = \boldsymbol{x} + \boldsymbol{v}_1 \\ z_3 = \boldsymbol{x} + \boldsymbol{v}_2 \\ z_4 = \boldsymbol{x} + \boldsymbol{v}_3 \\ z_5 = \boldsymbol{x} + \boldsymbol{v}_4 \end{cases}$$

Correlated !



Communication network

Estimation error of each Kalman Filter

Actual state of the formation flying s/c

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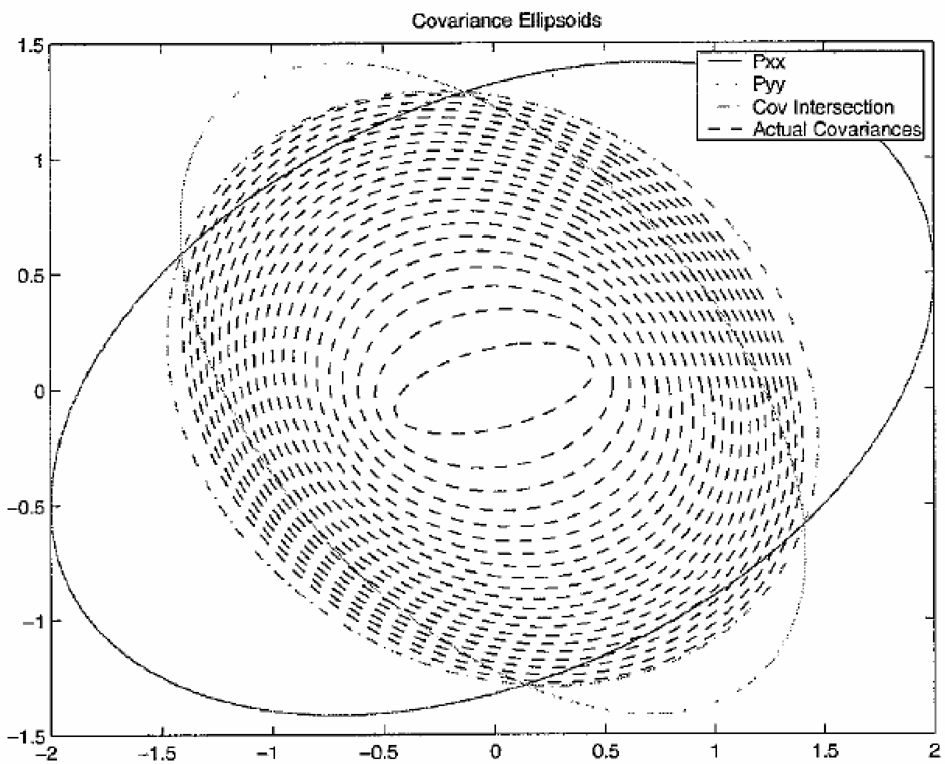
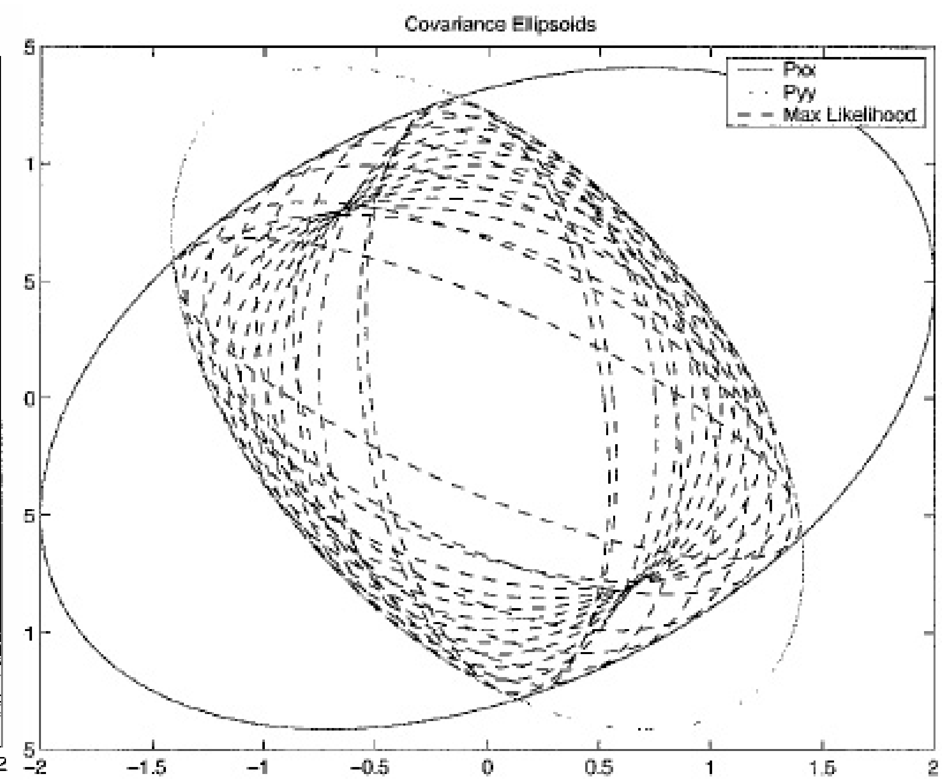
Considering $z=W_x x+W_y y$.

If Pxy is known → Maximum Likehood estimates minimize trace(Pzz). Intersection of the covariance ellipsoids of Pxx and Pyy gives the covariance ellipsoid of the Maximum Likehood estimator. **CI** provides an estimate and a covariance matrix whose ellipsoid encloses the intersection region without previous knowledge of cross-covariance,





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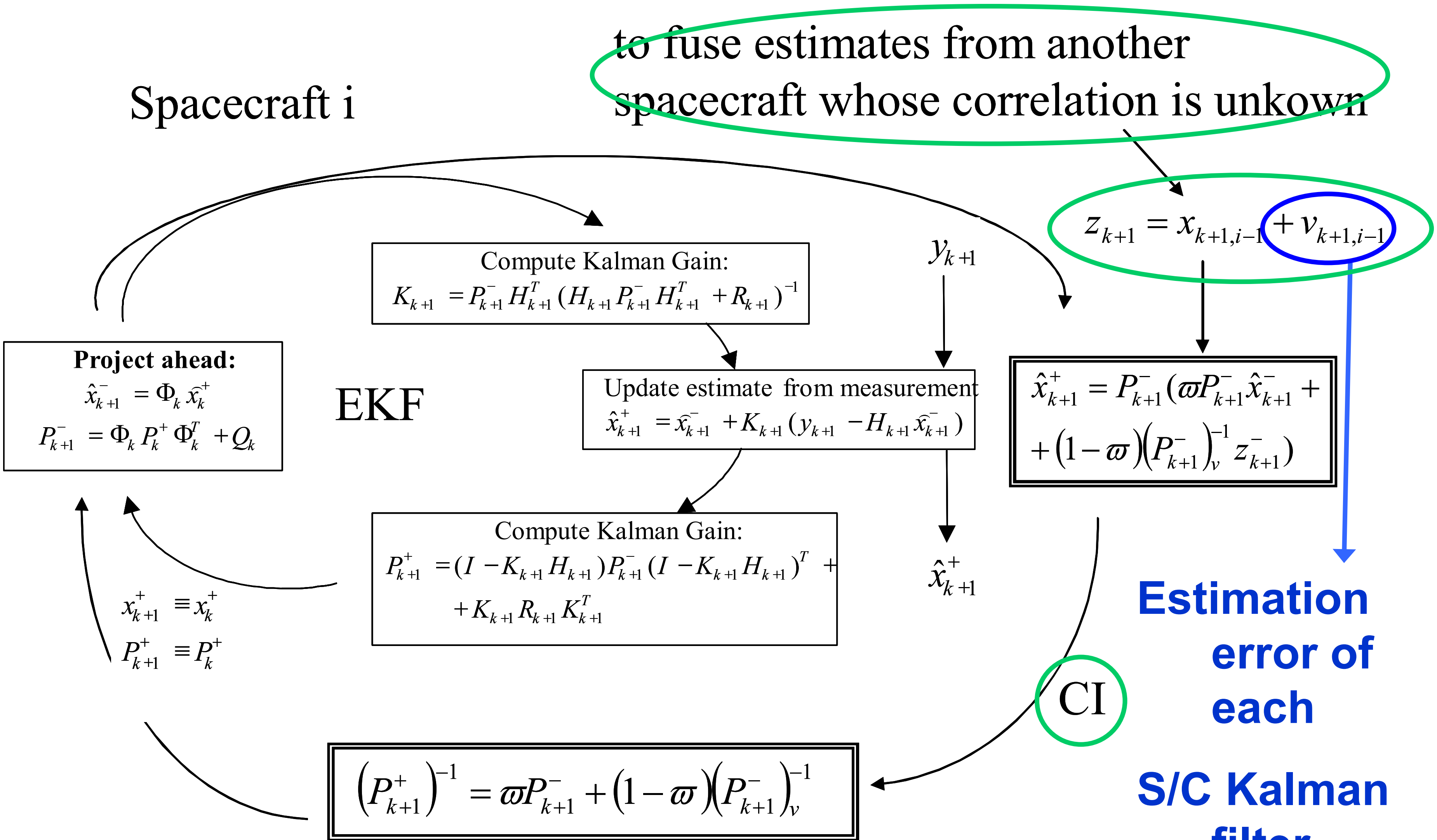
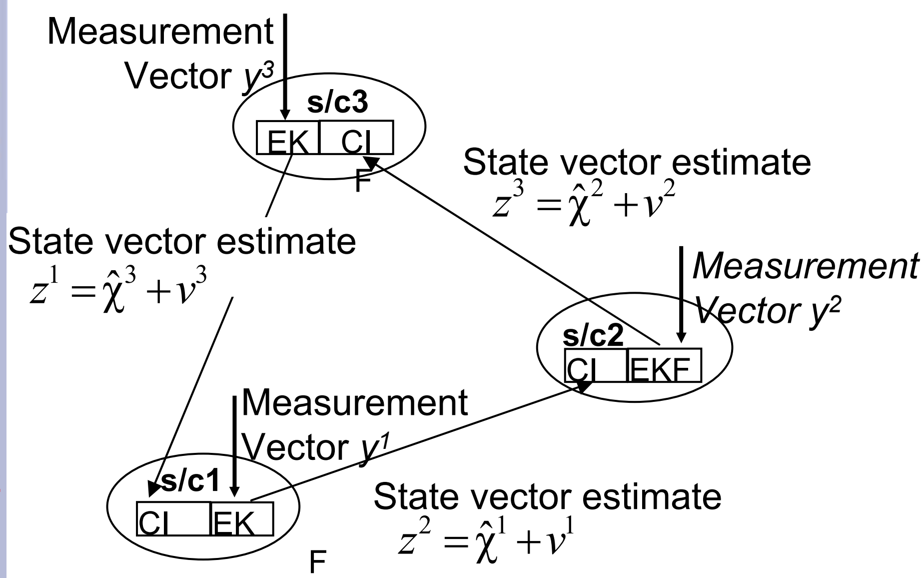
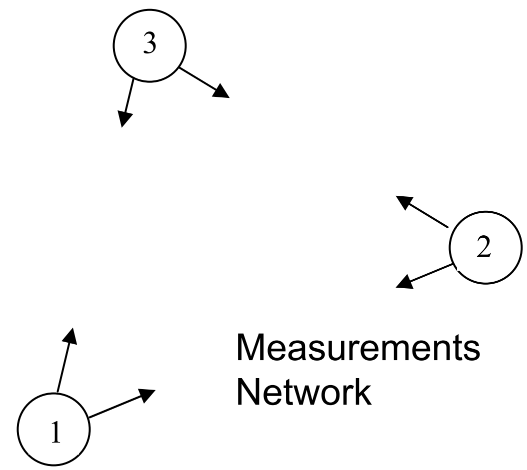
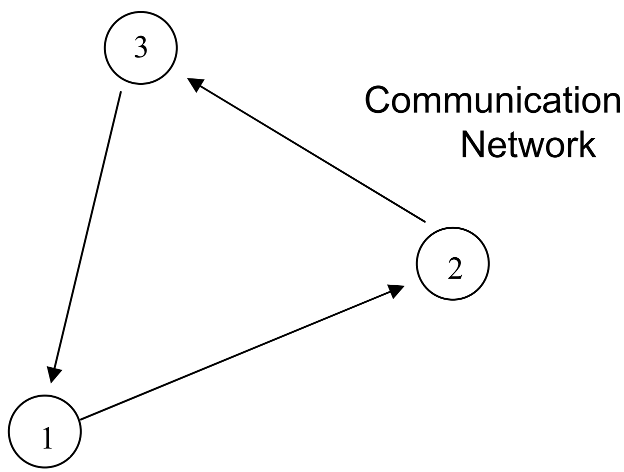


GNC for spacecraft formation → Navigation







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→ *Filtering with RF local measurements*

Measurements from RF system → $z^i(k) = y^i(k)$

EKF:

$$\hat{\chi}^{\mathrm{i}}_{\mathrm{PC}}(k/k+1) = \hat{\chi}^{\mathrm{i}}_{\mathrm{PC}}(k/k-1) + K^i(k)(z^i(k) - H^i(\hat{\chi}^{\mathrm{i}}_{\mathrm{PC}}(k/k-1)))$$

$$\boldsymbol{S}^{\boldsymbol{i}}(\boldsymbol{k}) = \boldsymbol{H}^{\boldsymbol{i}}(\boldsymbol{k})\boldsymbol{P}^{\boldsymbol{i}}(\boldsymbol{k}/\boldsymbol{k}-1)\left(\boldsymbol{H}^{\boldsymbol{i}}(\boldsymbol{k})\right)^{\boldsymbol{T}} + \boldsymbol{R}(\boldsymbol{k})$$

$$\boldsymbol{K}^{\boldsymbol{i}}(\boldsymbol{k}) = \boldsymbol{P}^{\boldsymbol{i}}(\boldsymbol{k}/\boldsymbol{k}-1)\boldsymbol{H}^{\boldsymbol{i}}(\boldsymbol{k})(\boldsymbol{S}^{\boldsymbol{i}}(\boldsymbol{k}))^{-1}$$

$$P^i(k/k+1) = \left(I - K^i(k)H^i(k)\right)P^i(k/k-1)\left(I - K^i(k)H^i(k)\right)^T + K^i(k)R^i(k)\left(K^i(k)\right)^T$$

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→ Fusion

Measurements from another spacecraft → $z^i(k) = \hat{\chi}^{i-1}_{PC}(k/k-1), P^{i-1}(k/k-1)$

CI: $0 \leq w \leq 1$

$$\hat{\chi}^{i}_{PC}(k/k-1) = P^i(k/k-1)\left(w\left(P^i(k/k-1)\right)^{-1}\hat{\chi}^{i}_{PC}(k/k-1) + (1-w)\left(P^{i-1}(k/k-1)\right)^{-1} z^i(k)\right)$$

$$\left(P^i(k/k+1)\right)^{-1} = w\left(P^i(k/k-1)\right)^{-1} + (1-w)\left(P^{i-1}(k/k-1)\right)^{-1}$$

$w=1$

$$\hat{\chi}^{i}_{PC}(k/k-1) = \hat{\chi}^{i}_{PC}(k/k-1)$$

$$P^i(k/k+1) = P^i(k/k-1)$$

New measurements from another spacecraft do not change the local estimates

$w=0$

$$\hat{\chi}^{i}_{PC}(k/k-1) = \left(P^{i-1}(k/k-1)\right)^{-1} z^i(k)$$

$$P^i(k/k+1) = P^{i-1}(k/k-1)$$

Locally, the estimates are neglected

EKF fusion:

$$\hat{\chi}^{i}_{PC}(k/k+1) = \hat{\chi}^{i}_{PC}(k/k-1) + K^i(k)\left(z^i(k) - H^i\hat{\chi}^{i}_{PC}(k/k-1)\right)$$

$$P^i(k/k+1) = \left(I - K^i(k)\right)P^i(k/k-1)\left(I - K^i(k)\right)^T + K^i(k)R^i(k)\left(K^i(k)\right)^T$$

$H^i = I$

$R^i = P^{i-1}(k/k-1)$

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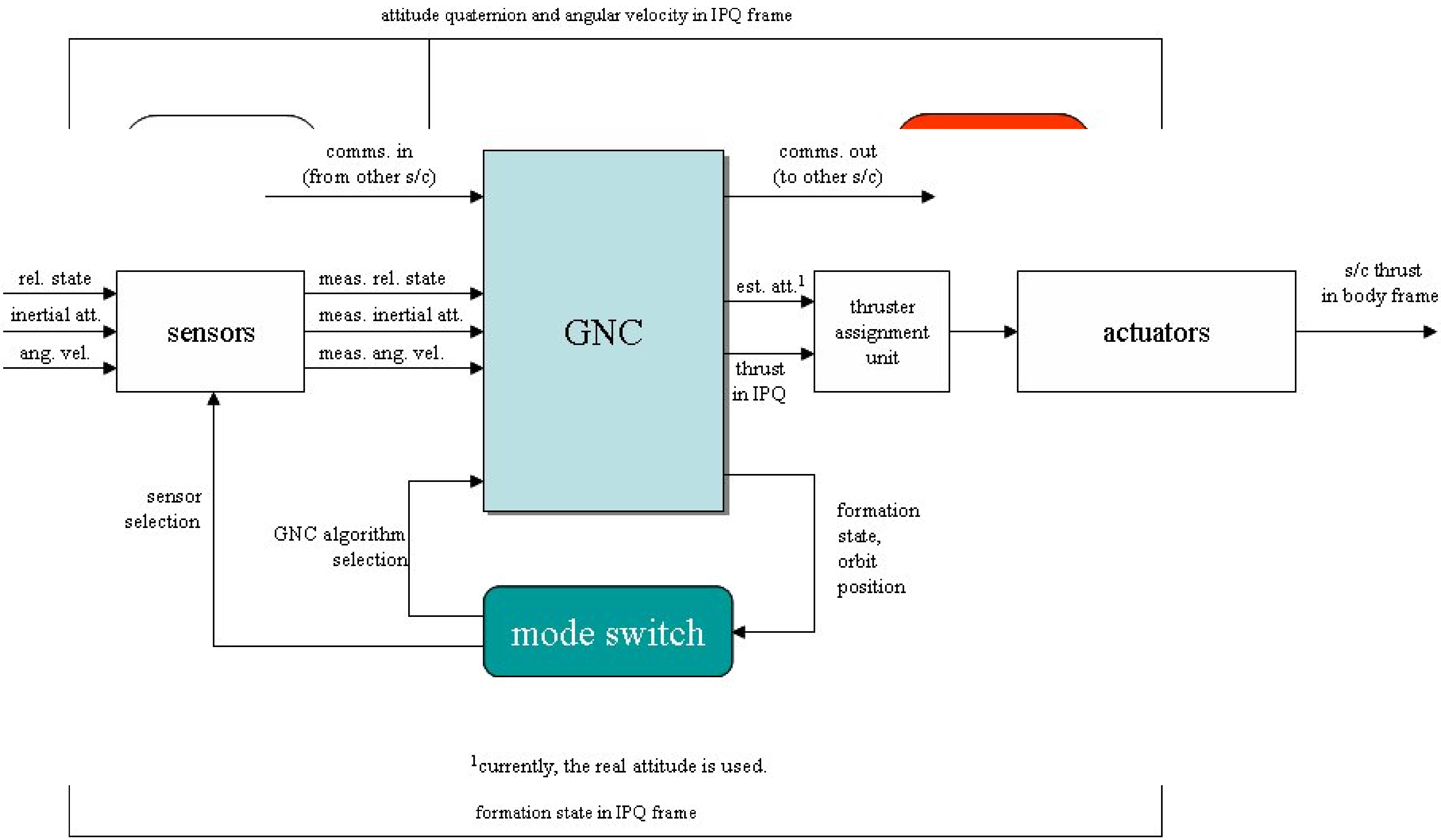
Formation Flying – Functional Engineering Simulator (FF-FES)



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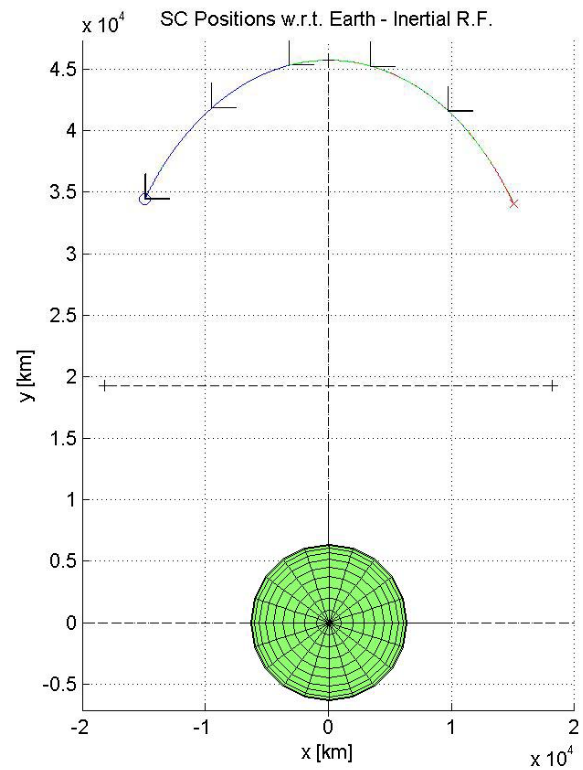
Closed Loop Estimation



	$\chi(\theta_0)$
$x_{12} = x_2 - x_1 [\mathrm{m}]$	3000.00
$y_{12} = y_2 - y_1 [\mathrm{m}]$	300.00
$z_{12} = z_2 - z_1 [\mathrm{m}]$	-864.00
$x_{32} = x_2 - x_3 [\mathrm{m}]$	2875.00
$y_{32} = y_2 - y_3 [\mathrm{m}]$	175.00
$z_{32} = z_2 - z_3 [\mathrm{m}]$	-3749.00
$x_{13} = x_3 - x_1 [\mathrm{m}]$	125.00
$y_{13} = y_3 - y_1 [\mathrm{m}]$	125.00
$z_{13} = z_3 - z_1 [\mathrm{m}]$	2885.00
$\dot{x}_{12} = \dot{x}_2 - \dot{x}_1 [\mathrm{m/s}]$	-0.04
$\dot{y}_{12} = \dot{y}_2 - \dot{y}_1 [\mathrm{m/s}]$	-0.04
$\dot{z}_{12} = \dot{z}_2 - \dot{z}_1 [\mathrm{m/s}]$	-0.04
$\dot{x}_{32} = \dot{x}_2 - \dot{x}_3 [\mathrm{m/s}]$	-0.02
$\dot{y}_{32} = \dot{y}_2 - \dot{y}_3 [\mathrm{m/s}]$	-0.02
$\dot{z}_{32} = \dot{z}_2 - \dot{z}_3 [\mathrm{m/s}]$	-0.06
$\dot{x}_{13} = \dot{x}_3 - \dot{x}_1 [\mathrm{m/s}]$	-0.02
$\dot{y}_{13} = \dot{y}_3 - \dot{y}_1 [\mathrm{m/s}]$	-0.02
$\dot{z}_{13} = \dot{z}_3 - \dot{z}_1 [\mathrm{m/s}]$	0.02

Navigation with

Sensors: RF only

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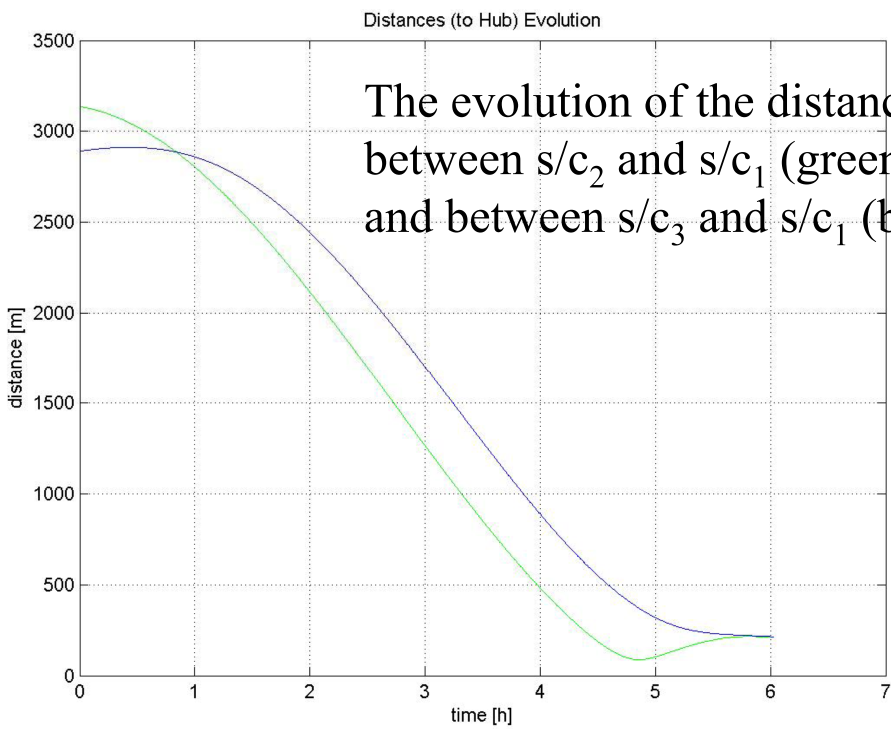
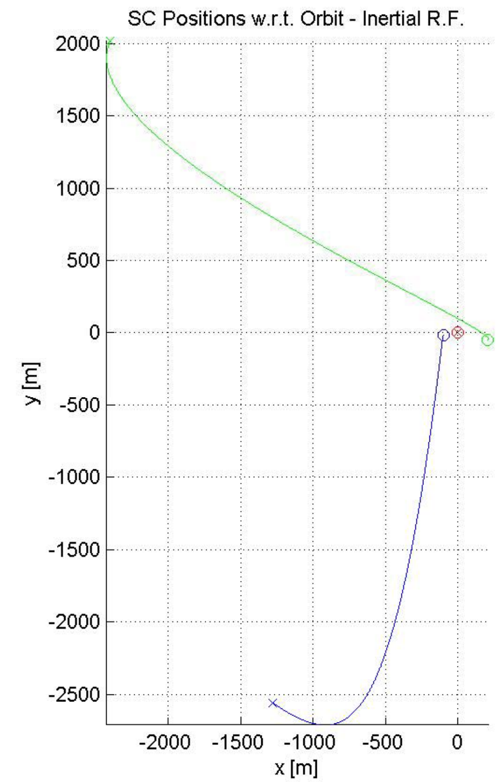
RESULTS obtained using DEIMOS' FF-FES simulator

EXAMPLE: *6 hours FAM mode centered around Apogee*



The evolution of the distances between s/c_2 and s/c_1 (green), and between s/c_3 and s/c_1 (blue)



Projection in *x-y* plane of the optimal relative trajectories in IPQ of s/c_2 (green) and s/c_3 (blue)

ERROR obtained = 15m for positions, 0.001m/s for velocities

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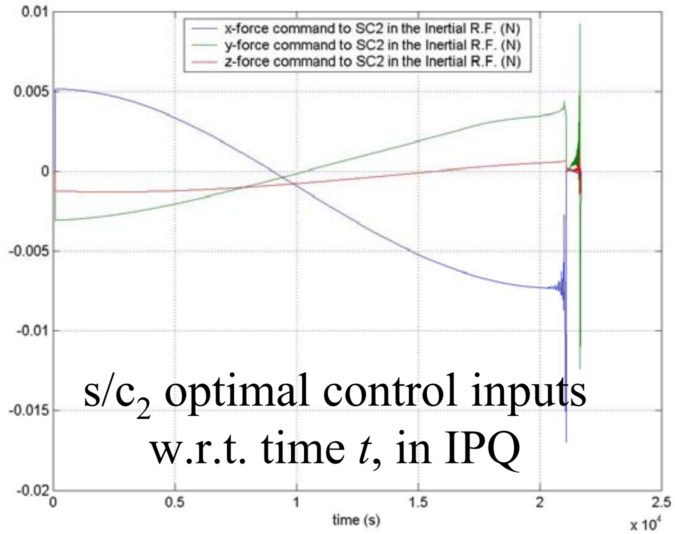
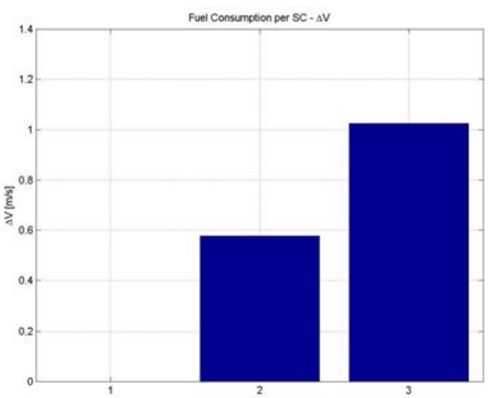
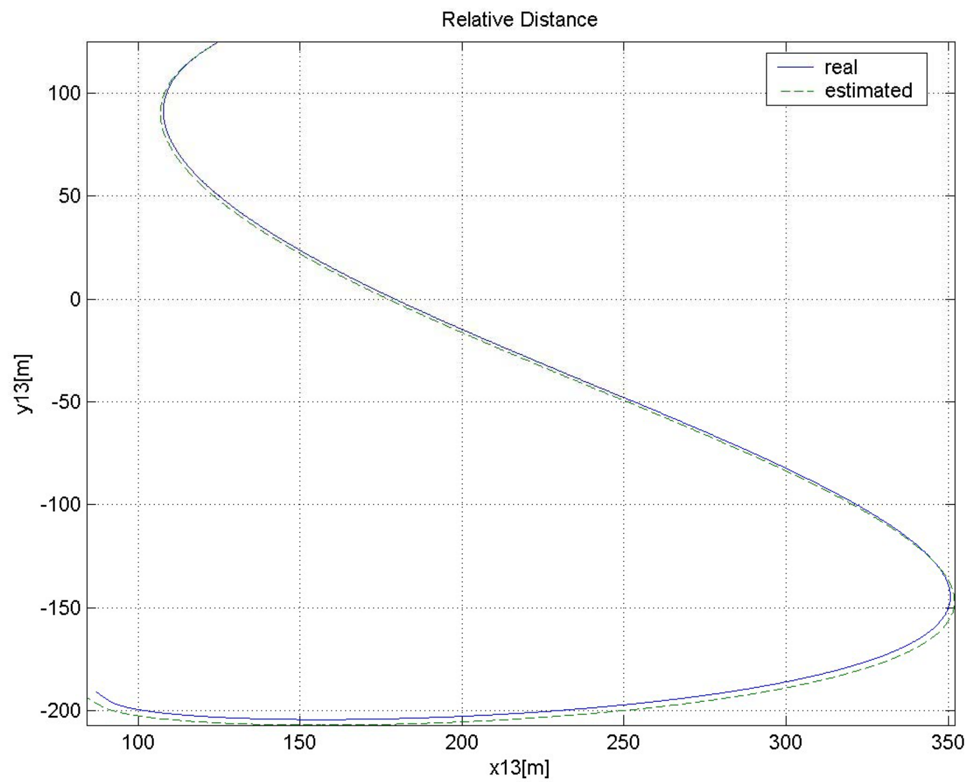


s/c_2 optimal control inputs w.r.t. time t, in IPQ



Fuel consumption



Real and estimated relative distances of the y component of s/c_1 w.r.t. x component of s/c_3.

For estimation only, the error is of order 10^{-4} m/s and maximum 10 meters for velocity and position respectively.

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Conclusions

- ***Guidance and Control***: we propose a model-based optimal trajectory planning algorithm.
  - Our guidance-oriented approach consists in regularly re-computing this algorithm.
  - This re-planning leads to trajectories that require less control effort during the trajectory tracking phase of the mission.
  - Condition to successfully achieve the FAM task: LARGE ENOUGH FAM duration.

- ***Navigation***: the formation state estimation is handled by a full-order decentralized estimator, based on the CI and EKF.
  - The EKF is used for local measurements, and for the measurements communicated by a predecessor spacecraft, in a method with no divergence troubles, i.e., the CI algorithm.