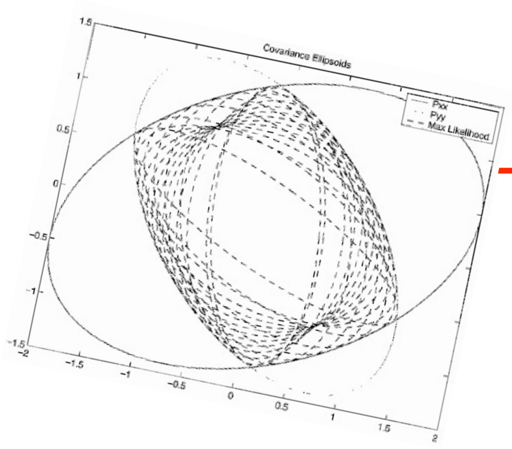
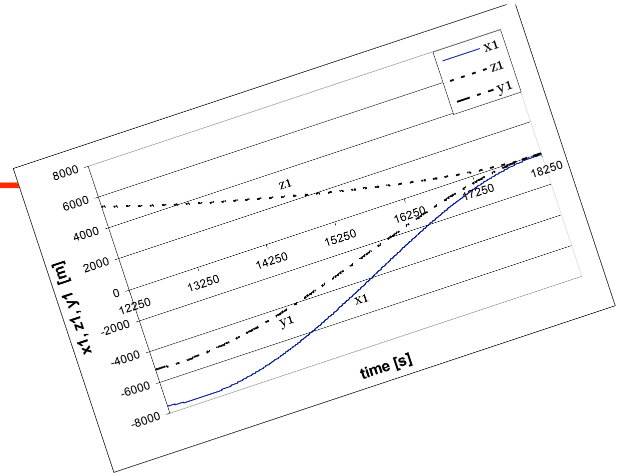
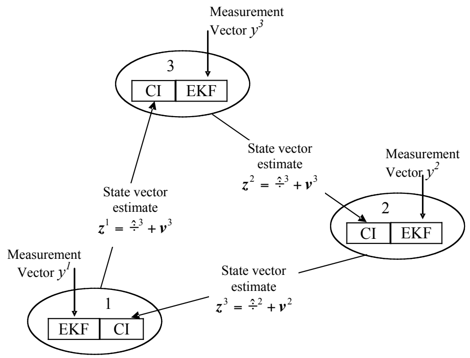


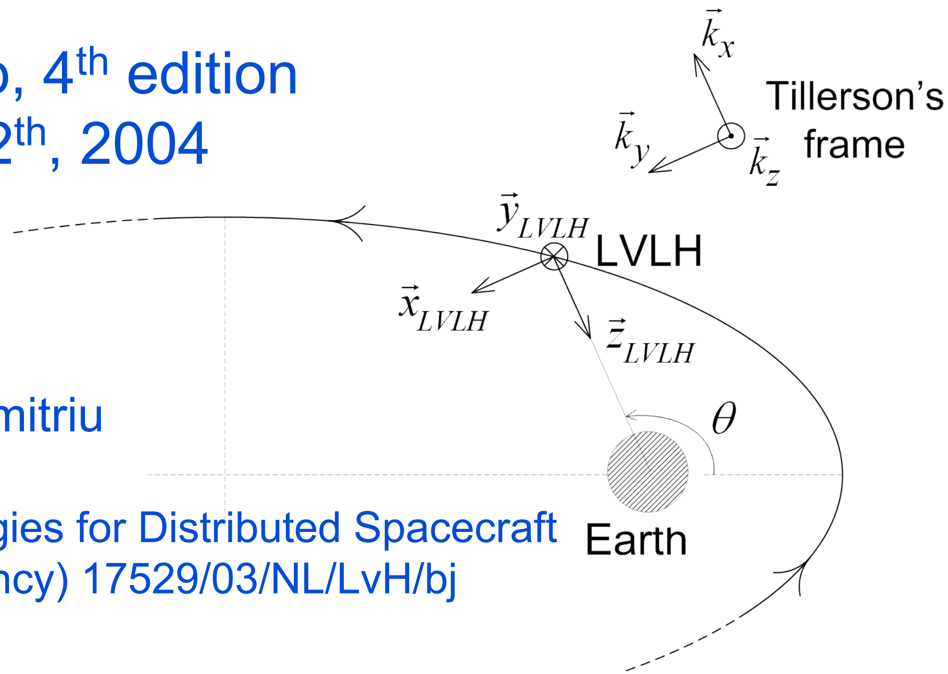


Optimal Trajectory Planning of Formation Flying Spacecraft



ISLab Workshop, 4th edition

November 12th, 2004



Dan Dumitriu

Formation Estimation Methodologies for Distributed Spacecraft

ESA (European Space Agency) 17529/03/NL/LvH/bj

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Plan of the presentation

- **Context of the problem**
- **Relative Dynamics for Eccentric Orbits**
- **Formation Initialization Optimal Control Problem**
- **Results**
- **Conclusions & questions**

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Current and/or future trend in space science missions: the usage of several spacecraft flying in formation, rather than using monolithic platforms

- higher accuracy in Earth and extra solar planetary observations
- higher region coverage when monitoring science data

- ***ESA project on Formation Flying of 3 Spacecraft in Geostationary Transfer Orbit (GTO) – phase II***

- DEIMOS Engenharia → FF-FES Matlab/Simulink simulator
- ISR/IST (project manager Pedro Lima) → reliable Guidance, Navigation and Control algorithms, implemented as S-functions in the simulator

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Guidance&Control goal during the Formation Acquisition Mode (FAM) :

- Bring the 3 spacecraft
  - → from an initial *randomly* dispersed disposition (at t_1) within a sphere of 8km in diameter
  - → to a desired final disposition at t_2, which is a tight formation = distances between TF1 (telescope flyer) and Hub (master satellite) and between TF2 and Hub of 250m, with an aperture angle of the formation of 120º
- by minimizing the fuel spent of all spacecraft and by avoiding collisions

Orbital parameters of the GTO orbit

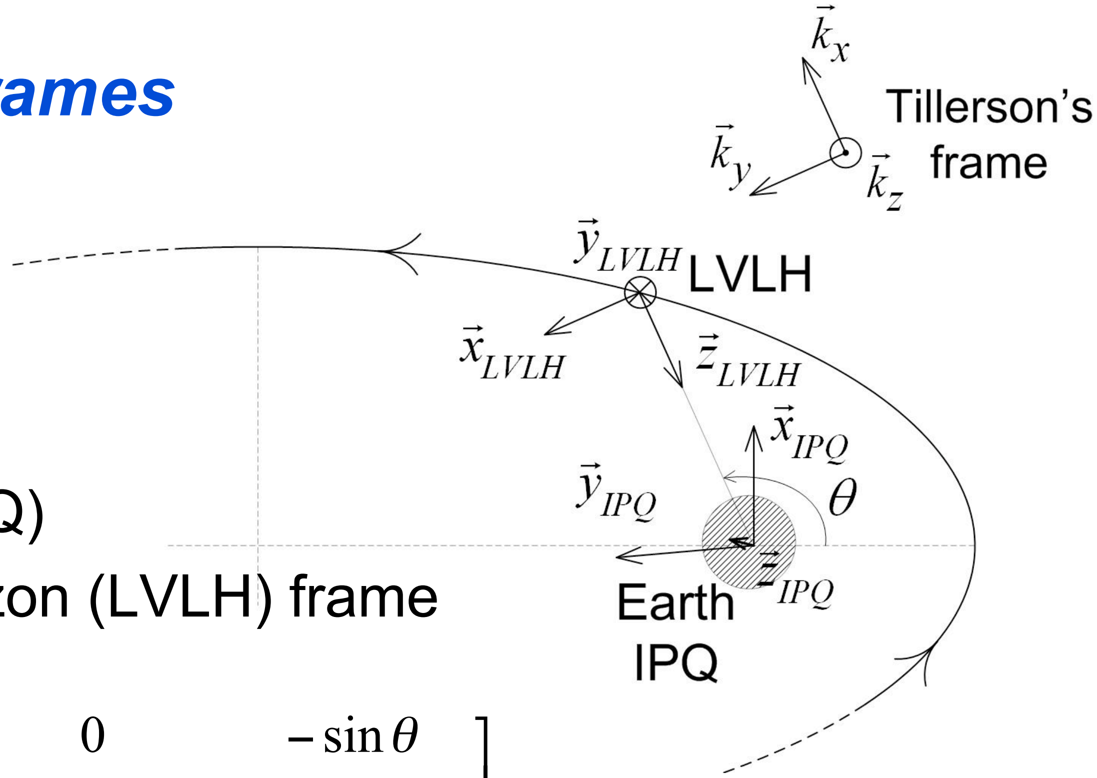
- Semi-major axis: $a = 26624.137\text{km}$
- Eccentricity: $e = 0.73039$
- RAAN: $\Omega = 0°$
- Inclination: $i = 7°$
- Argument of perigee: $\omega = -90°$
- True anomaly θ

$$t - t_p = \frac{1}{n}\left[2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right) - \frac{e\sqrt{1-e^2}\sin\theta}{1+e\cos\theta}\right]$$

Other derived parameters:

- the natural frequency of the reference orbit: $n = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{G \cdot m_{Earth}}{a^3}}$
- the period of the orbit: $T = \frac{2\pi}{n} = 43233.88\text{s} = 12\text{h } 33.88\text{s}$

Reference frames



- Inertial Planet Frame (IPQ)
- Local Vertical Local Horizon (LVLH) frame

$$\vec{w}_{IPQ} = \mathbf{R}\,\vec{w}_{LVLH} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \cos 7^{o}\sin\theta & \sin 7^{o} & \cos 7^{o}\cos\theta \\ \sin 7^{o}\sin\theta & -\cos 7^{o} & \sin 7^{o}\cos\theta \end{bmatrix} \vec{w}_{LVLH}$$

$$\vec{w}_{LVLH} = \mathbf{R}^{T}\,\vec{w}_{IPQ} \quad (\textit{transformations valid for position vectors})$$

$$\dot{\vec{w}}_{LVLH} = \dot{\mathbf{R}}^{T}\,\vec{w}_{IPQ} + \mathbf{R}^{T}\,\dot{\vec{w}}_{IPQ} \quad (\textit{for velocity vectors})$$

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θ-varying relative dynamics equation (in LVLH)

- In-plane motion of i th spacecraft

$$\frac{d}{d\theta}\begin{bmatrix} x_i \\ x_i' \\ z_i \\ z_i' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{e\cos\theta}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} & \frac{-2e\sin\theta}{1+e\cos\theta} & 2 \\ 0 & 0 & 0 & 1 \\ \frac{2e\sin\theta}{1+e\cos\theta} & -2 & \frac{3+e\cos\theta}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} \end{bmatrix}\begin{bmatrix} x_i \\ x_i' \\ z_i \\ z_i' \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_{i,x} + \sum w_{i,x} \\ u_{i,z} + \sum w_{i,z} \end{bmatrix}$$

- Out-of-plane motion

$$\frac{d}{d\theta}\begin{bmatrix} y_i \\ y_i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{1+e\cos\theta} & \frac{2e\sin\theta}{1+e\cos\theta} \end{bmatrix}\begin{bmatrix} y_i \\ y_i' \end{bmatrix} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(u_{i,y} + \sum w_{i,y}\right)$$

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Considered perturbations $\vec{w}_{i,J_2},\ \vec{w}_{i,Moon},\ \vec{w}_{i,Sun}$

- *Perturbation due to J_2 effect (in IPQ)*

$$w_{i,J_2,x} = \mu A_{J_2} \frac{15 r_x \left(\|\vec{r}\|^2 - 7 r_z^2\right)\langle \vec{\rho}_i, \vec{r}\rangle + 15 r_z \|\vec{r}\|^2 (r_z x_i + 2 r_x z_i) - 3\|\vec{r}\|^4 x_i}{\|\vec{r}\|^7 \left(\|\vec{r}\|^2 + 12(r_x x_i + r_y y_i + r_z z_i)\right)}$$

$$w_{i,J_2,y} = \ldots$$

$$w_{i,J_2,z} = \ldots \qquad \vec{\rho}_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T, \quad \vec{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T$$

- *Third-body gravitational perturbation (in IPQ)*

$$\vec{w}_{i,Moon} = -\mu_{Moon} \frac{\|\vec{r} - \vec{r}_{Moon}\|^2 \vec{\rho}_i - 3\langle \vec{\rho}_i, \vec{r} - \vec{r}_{Moon}\rangle (\vec{r} - \vec{r}_{Moon})}{\|\vec{r} - \vec{r}_{Moon}\|^3 \left(\|\vec{r} - \vec{r}_{Moon}\|^2 + 3\langle \vec{\rho}_i, \vec{r} - \vec{r}_{Moon}\rangle\right)}$$

analogously for $\vec{w}_{i,Sun}$

Other perturbations: atmospheric drag, solar radiation pressure, micrometeoroids

Optimal Control Problem during FAM

- State equations = Relative dynamics equations (linearized in what concerns the gravitational accelerations, but slightly non-linear because of perturbations terms)
- 2-boundary conditions (initial and final conditions)
- Limitations concerning the control inputs (actuators saturation)
- The cost function to be minimized (takes into account both fuel consumption & collision avoidance)

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State equations :

- by putting together the relative dynamics equations

$$\frac{d[\mathbf{X}(\theta)]}{d\theta} = \mathbf{A}(\theta)\mathbf{X}(\theta) + \mathbf{B}(\theta)\left[\mathbf{U}(\theta) + \mathbf{W}(\theta)\right]$$

$$\text{where} \quad \mathbf{X} = \left[x_1 \; x_1' \; z_1 \; z_1' \; y_1 \; y_1' \; x_2 \; x_2' \; z_2 \; z_2' \; y_2 \; y_2' \; x_3 \; x_3' \; z_3 \; z_3' \; y_3 \; y_3'\right]^T$$

$$\mathbf{U} = \left[u_{1,x} \; u_{1,z} \; u_{1,y} \; u_{2,x} \; u_{2,z} \; u_{2,y} \; u_{3,x} \; u_{3,z} \; u_{3,y}\right]^T$$

Two-boundary conditions :

- FAM takes place between θ_1 and θ_2 (t_1 and t_2)
- considered FAM duration: $\Delta t_{12} = t_2 - t_1 = 4\text{h}$ (large enough in order not to overload the actuators)

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Initial conditions (at θ_1)

- initial *randomly* dispersed disposition within a sphere of 8km in diameter (1km for the moment, the dimensioning of the problem being still in a study phase)
- velocities are very small, as after dispenser we have a cancel relative velocity mode

$$\text{positions at } \theta_1: \quad Hub\begin{cases} x_1 = -498.48\text{m} \\ z_1 = -287.66\text{m} \\ y_1 = 101.14\text{m} \end{cases} \quad TF1\begin{cases} x_2 = 495.21\text{m} \\ z_2 = -292.73\text{m} \\ y_2 = 45.17\text{m} \end{cases} \quad TF2\begin{cases} x_3 = 27.04\text{m} \\ z_3 = 580.24\text{m} \\ y_3 = 27.38\text{m} \end{cases}$$

$$\text{velocities at } \theta_1: \quad Hub\begin{cases} \dot{x}_1 = 15.046\frac{\text{mm}}{\text{s}} \\ \dot{z}_1 = -4.152\frac{\text{mm}}{\text{s}} \\ \dot{y}_1 = 9.071\frac{\text{mm}}{\text{s}} \end{cases} \quad TF1\begin{cases} \dot{x}_2 = -39.905\frac{\text{mm}}{\text{s}} \\ \dot{z}_2 = -38.842\frac{\text{mm}}{\text{s}} \\ \dot{y}_2 = -40.44\frac{\text{mm}}{\text{s}} \end{cases} \quad TF2\begin{cases} \dot{x}_3 = 14.251\frac{\text{mm}}{\text{s}} \\ \dot{z}_3 = 33.890\frac{\text{mm}}{\text{s}} \\ \dot{y}_3 = 19.758\frac{\text{mm}}{\text{s}} \end{cases}$$

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Final conditions (at θ_2)

- the desired final disposition is a tight formation = distances between TF1 and Hub and between TF2 and Hub of 250m, with an aperture angle of the formation of 120º

- These formation conditions are provided by Deimos, as a result of an optimal design. After FAM, near Perigee, we need to precisely maintain this formation during a 2h observation experiment. The goal of the optimal design was to minimize the control inputs for maintaining the formation during these 2h.

$$\textit{positions at } \theta_2: \quad Hub\begin{cases} x_1 = 0.0\text{m} \\ z_1 = 0.0\text{m} \\ y_1 = 0.0\text{m} \end{cases} \quad TF1\begin{cases} x_2 = -248.174\text{m} \\ z_2 = 30.154\text{m} \\ y_2 = 0.0\text{m} \end{cases} \quad TF2\begin{cases} x_3 = 124.087\text{m} \\ z_3 = -15.077\text{m} \\ y_3 = 216.506\text{m} \end{cases}$$

$$\textit{velocities at } \theta_2: \quad Hub\begin{cases} \dot{x}_1 = 0.0\frac{\text{mm}}{\text{s}} \\ \dot{z}_1 = 0.0\frac{\text{mm}}{\text{s}} \\ \dot{y}_1 = 0.0\frac{\text{mm}}{\text{s}} \end{cases} \quad TF1\begin{cases} \dot{x}_2 = -4.026\frac{\text{mm}}{\text{s}} \\ \dot{z}_2 = -11.030\frac{\text{mm}}{\text{s}} \\ \dot{y}_2 = -0.1548\frac{\text{mm}}{\text{s}} \end{cases} \quad TF2\begin{cases} \dot{x}_3 = 2.211\frac{\text{mm}}{\text{s}} \\ \dot{z}_3 = 4.898\frac{\text{mm}}{\text{s}} \\ \dot{y}_3 = -0.4204\frac{\text{mm}}{\text{s}} \end{cases}$$

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Control inputs limitations

$$u_{\min} \le |U_j| \le u_{\max}, \quad for\ j = 1,\ldots,9 \quad (U_j = u_{1,x} / u_{1,z} / u_{1,y} / u_{2,x} / \ldots)$$

$$u_{\min} = 0.1\mu\text{N}$$

$$u_{\max} = 15\text{mN}$$ (important value for dimensioning of the problem)

- **Remark:** The control inputs limitations are AUTOMATICALLY taken into account in the right side of the state dynamics equations, when integrating numerically these equations. We just don't allow control inputs to exceed the limitations.

Example: if $U_j > u_{max}$, then we impose $U_j = u_{max}$

if $U_j < -u_{max}$, then we impose $U_j = -u_{max}$

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Cost function to be minimized

- The cost function (performance index) takes into account BOTH fuel spent and collision avoidance :

$$J = \int_{\theta_1}^{\theta_2} L\big(\mathbf{X}(\theta), \mathbf{U}(\theta), \theta\big)\, d\theta = J_{fuel} + J_{avoidance} =$$

$$= \int_{\theta_1}^{\theta_2} \sum_{j=1}^{9} U_j^2\, d\theta \; + \int_{\theta_1}^{\theta_2} \Big[\delta_{12}(\rho_{12} - \rho_{\min})^2 + \delta_{13}(\rho_{13} - \rho_{\min})^2 + \delta_{23}(\rho_{23} - \rho_{\min})^2\Big] d\theta$$

$\rho_{12} = relative\ \ distance\ \ between\ \ Hub\ \ and\ \ TF1$

$\rho_{\min} = 40\,[m]$

$$weighting\ \ coefficient\ \ \delta_{12} = \begin{cases} 0 & if\ \ \rho_{12} \geq \rho_{\min} \\ \delta^0 & if\ \ \rho_{12} < \rho_{\min} \end{cases}$$

- $we\ \ choose\ \ \delta^0\ \ for\ \ a\ \ good\ \ ponderation\ \ between\ \ J_{fuel}\ \ and\ \ J_{avoidance}$

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Pontryagin Maximum Principle (PMP) formulation :

Hamiltonian: $H(\mathbf{X},\mathbf{U},\theta) = L(\mathbf{X},\mathbf{U},\theta) + \ddot{\mathbf{E}}^T \mathbf{f}(\mathbf{X},\mathbf{U},\theta) = L(\mathbf{X},\mathbf{U},\theta) + \sum_{k=1}^{18} \lambda_k f_k(\mathbf{X},\mathbf{U},\theta)$

Φ State equations: $$\frac{dX_i}{d\theta} = \frac{\partial H}{\partial \lambda_i} = f_i, \quad for \;\; i = 1, \mathrm{K}\;, 18$$

Φ Co-state equations:
(λ_i - adjoint variables) $$\frac{d\lambda_i}{d\theta} = -\frac{\partial H}{\partial X_i} = -\frac{\partial L}{\partial X_i} - \sum_{k=1}^{18} \frac{\partial f_k}{\partial X_i}\lambda_k$$

PMP: The control inputs, which satisfy, for $\theta_1 \leq \theta \leq \theta_2$, the *stationarity conditions*:

$$0 = \frac{\partial H}{\partial U_j} = \frac{\partial L}{\partial U_j} + \sum_{k=1}^{18} \frac{\partial f_k}{\partial U_j}\lambda_k, \quad for \;\; j = 1, \mathrm{K}\;, 9$$

are the optimal control inputs, the corresponding trajectory being optimal as well !

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1) **State equations**: the relative dynamics equations for all 3 spacecraft

2) **Co-state equations :**

$$\frac{d\lambda_1}{d\theta} = -\frac{\partial L}{\partial X_1} - \left[\frac{e\cos\theta}{1+e\cos\theta} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\frac{\partial w_{i,x}}{\partial X_1}\right]\lambda_2 - $$

$$-\left[\frac{2e\sin\theta}{1+e\cos\theta} + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\frac{\partial w_{i,z}}{\partial X_1}\right]\lambda_4 - \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\frac{\partial w_{i,y}}{\partial X_1}\lambda_6 ,$$

$$where \ \frac{\partial L}{\partial X_1} = 2\left[\delta_{12}\left(1-\frac{\rho_{min}}{\rho_{12}}\right)(X_1 - X_7) + \delta_{13}\left(1-\frac{\rho_{min}}{\rho_{13}}\right)(X_1 - X_{13})\right]$$

$$\frac{d\lambda_2}{d\theta} = -\lambda_1 - \frac{2e\sin\theta}{1+e\cos\theta}\lambda_2 + 2\lambda_4 ,$$

$$\frac{d\lambda_3}{d\theta} = \text{K}$$

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3) ***Stationarity conditions*** **:**

$$0 = 2U_1 + \frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\lambda_2 \quad \Rightarrow \quad U_1 = -\frac{1}{2}\frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\lambda_2 \,,$$

$$U_2 = -\frac{1}{2}\frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\lambda_4 \,, \quad \ldots$$

By summarizing : $U_j = -\frac{1}{2}\frac{(1-e^2)^3}{(1+e\cos\theta)^4 n^2}\lambda_{2j} \,, \quad for \;\; j = 1,\ldots,9$

So, by means of the stationarity conditions, the optimal control inputs U_j are directly linked to the adjoint variables λ_i.

- Advantage of PMP over Linear Programming: PMP works also with NON-LINEAR state equations, so perturbations can be taken into account

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Iterative "shooting method" *in order to solve* ***the differential two-boundary equations system***

$$\begin{cases} \dfrac{d\mathbf{X}}{d\theta} = \mathbf{f}(\mathbf{X},\mathbf{U},\theta), \\ \dfrac{d\ddot{\mathbf{E}}}{d\theta} = -\left(\dfrac{\partial H}{\partial \mathbf{X}}\right)^T = \mathbf{g}(\mathbf{X},\mathbf{U},\theta) \end{cases}$$

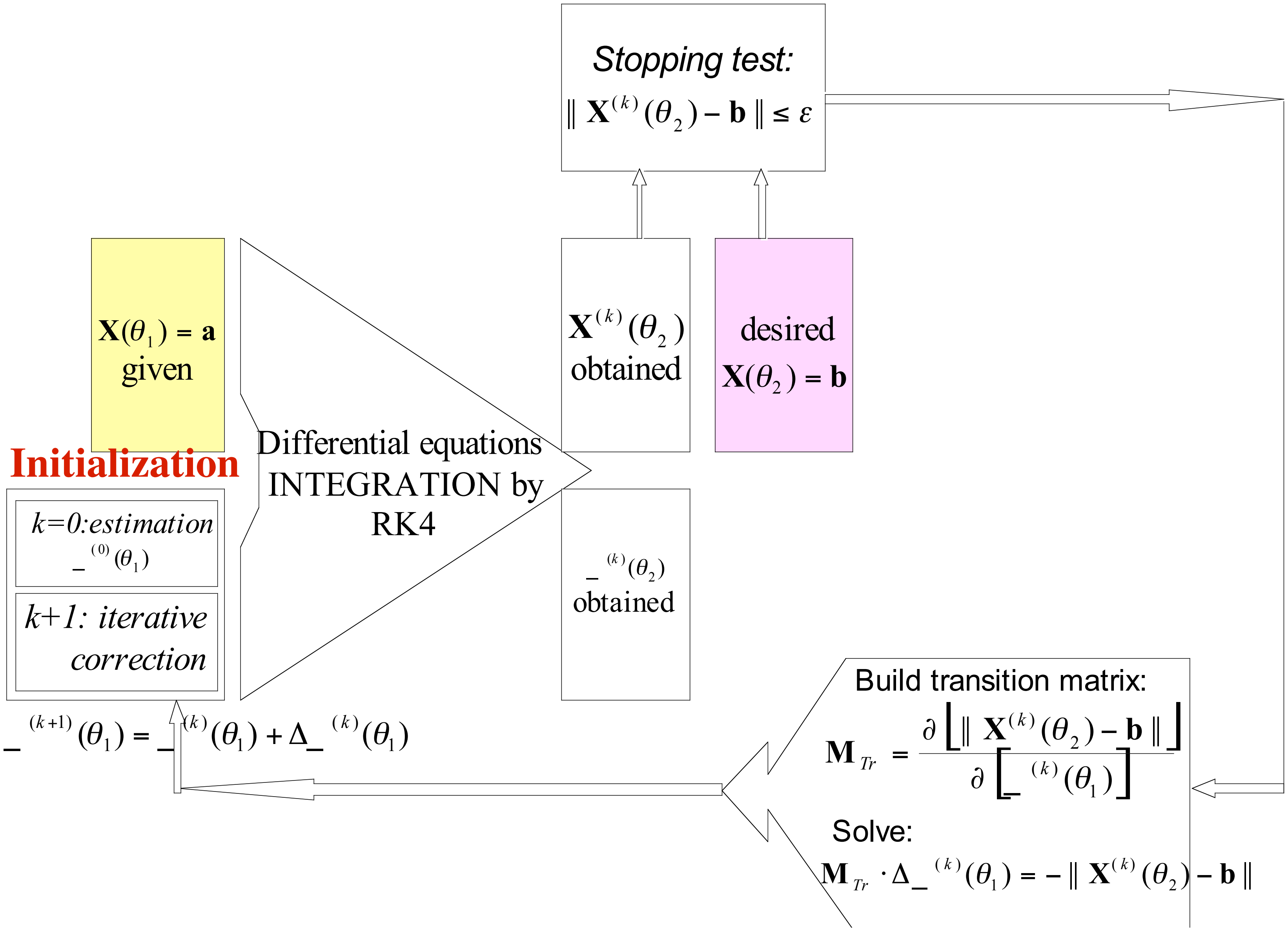
$$\mathbf{X}(\theta_1) = \mathbf{a} \quad and \quad \mathbf{X}(\theta_2) = \mathbf{b}$$

- *no initial or final condition for* $\ddot{\mathbf{E}}$

1. First iteration k=0: INITIALIZING the adjoint variables $\Lambda^{(0)}(\theta_1)$
2. At iteration k : CORRECTION of the initial adjoint variables $\Lambda^{(k)}(\theta_1)$, in order that, after integration of the differential equations above between θ_1 and θ_2, the following stopping test to be satisfied: $$\|\mathbf{X}^{(k)}(\theta_2) - \mathbf{b}\| \le \varepsilon$$

Once the test satisfied, $\mathbf{X}^{(k)}(\theta)$ is the optimal trajectory and $\mathbf{U}^{(k)}(\theta)$ are the optimal control inputs, for $\theta_1 \le \theta \le \theta_2$!

Guidance&Control → Iterative shooting method



Reliable Initialization of the Shooting Method based on the PMP Formulation

- The differential state equations (without perturbations) are:

$$\frac{d\mathbf{X}_i}{d\theta} = \mathbf{A}_i(\theta)\mathbf{X}_i(\theta) + \mathbf{B}_i(\theta)\mathbf{U}_i(\theta), \qquad \mathit{for} \;\; i = 1,2,3$$

$$or \quad \left.\frac{d\mathbf{X}_i}{d\theta}\right|_{\theta_k} = \mathbf{A}_i(\theta_k)\mathbf{X}_i(\theta_k) + \mathbf{B}_i^{\Lambda}(\theta_k)\ddot{\mathbf{E}}_i(\theta_k)$$

$$\Rightarrow \mathbf{X}_i(k+1) = \left[(\delta\theta)\mathbf{A}_i(k) + \mathbf{I}_6\right]\mathbf{X}_i(k) + \left[(\delta\theta)\mathbf{B}_i^{\Lambda}(k)\right]\ddot{\mathbf{E}}_i(k)$$

Finally, the recurrent expression of the state variables is:

$$\mathbf{X}_i(k+1) = \overline{\mathbf{A}}_i(k)\mathbf{X}_i(k) + \overline{\mathbf{B}}_i(k)\ddot{\mathbf{E}}_i(k)$$

- *Recurrent expression for the adjoint variables (co-state) vector:*

$$\ddot{\mathbf{E}}_i(k+1) = \overline{\mathbf{C}}_i(k)\ddot{\mathbf{E}}_i(k)$$

$\mathbf{X}_i(k+1)$ *expressed directly as function of* $\mathbf{X}_i(0)$ *and* $\mathbf{\Lambda}_i(0)$:

$$\mathbf{X}_i(k+1) = \mathbf{P}_i(k)\mathbf{X}_i(0) + \mathbf{Q}_i(k)\ddot{\mathbf{E}}_i(0)$$

$$\ddot{\mathbf{E}}_i(k+1) = \mathbf{N}_i(k)\ddot{\mathbf{E}}_i(0)$$

Recurrent sequence:

1. Φ $\mathbf{P}_i(0) = \overline{\mathbf{A}}_i(0), \quad \mathbf{Q}_i(0) = \overline{\mathbf{B}}_i(0), \quad \mathbf{N}_i(0) = \overline{\mathbf{C}}_i(0)$

2. Φ FOR *k*=1 TO *n*-1

$$\mathbf{P}_i(k) = \overline{\mathbf{A}}_i(k)\mathbf{P}_i(k-1)$$

$$\mathbf{Q}_i(k) = \overline{\mathbf{A}}_i(k)\mathbf{Q}_i(k-1) + \overline{\mathbf{B}}_i(k)\mathbf{N}_i(k-1)$$

$$\mathbf{N}_i(k) = \overline{\mathbf{C}}_i(k)\mathbf{N}_i(k-1)$$

$$\mathbf{X}_i(\theta_{k=n}) = \mathbf{P}_i(n-1)\,\hat{\dot{\mathbf{X}}}_i(\theta_{k=0}) + \mathbf{Q}_i(n-1)\ddot{\mathbf{E}}_i(0)$$

$$or \quad \mathbf{Q}_i(n-1)\ddot{\mathbf{E}}_i(0) = \mathbf{X}_i(\theta_2) - \mathbf{P}_i(n-1)\,\hat{\dot{\mathbf{X}}}_i(\theta_1)$$

- ***Algebraic system of 6 linear equations*** *(unknowns* $\mathbf{\Lambda}_i(0)$*), easily solved by using the Gauss elimination method.*
- PERTURBATIONS not considered ⇒ Only linearized expressions of the perturbations can be taken into account

Closed-loop LINEAR CONTROLLER

- This Initialization method for the PMP based shooting algorithm is nothing else than an <u>closed-loop linear controller</u> ⇐ we obtain the optimal control inputs, by using the stationarity conditions

- In practice, we execute this linear controller every 100s, and for the next 100s we apply the optimal control inputs just computed

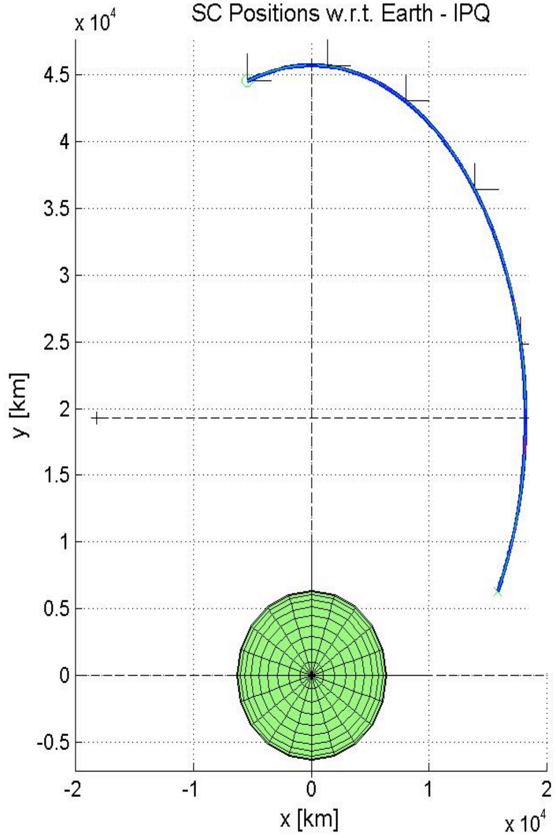
RESULTS obtained with the DEIMOS' FF-FES simulator

- The shooting method based on the PMP formulation is implemented (as Matlab/Simulink S-function written in C code), in order to find the optimal trajectory between θ_1 and θ_2
- The adjoint variables INITIALIZATION method is already programmed / the implementation of the equivalent CLOSED-LOOP LINEAR CONTROLLER nearly done
- Up to now, the simulations are run with perturbations disabled

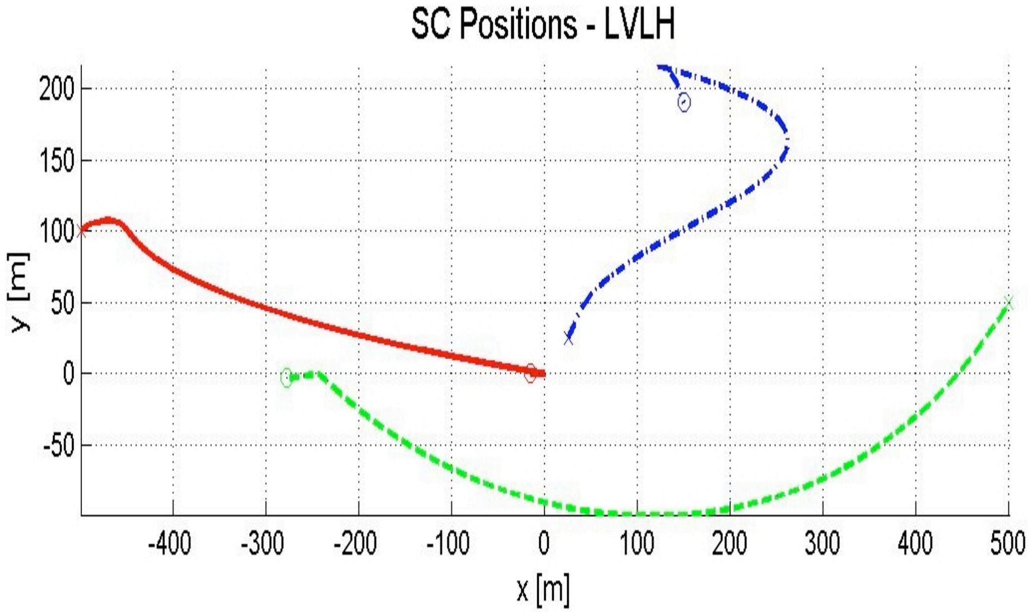
Simulation conditions

Final conditions ⇔ triangle with aperture angle of 120º, and with distances of 250m between TF1 and the hub and between TF2 and the hub

Guidance&Control → Matlab/Simulink simulator results

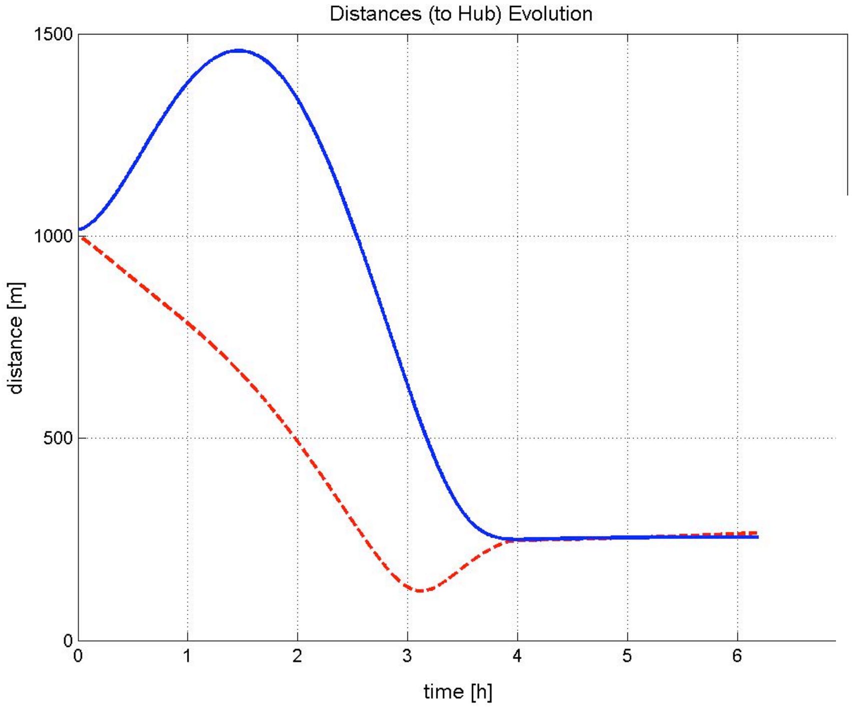


View from above the orbital plane of the 3 spacecraft positions w.r.t Earth, in IPQ

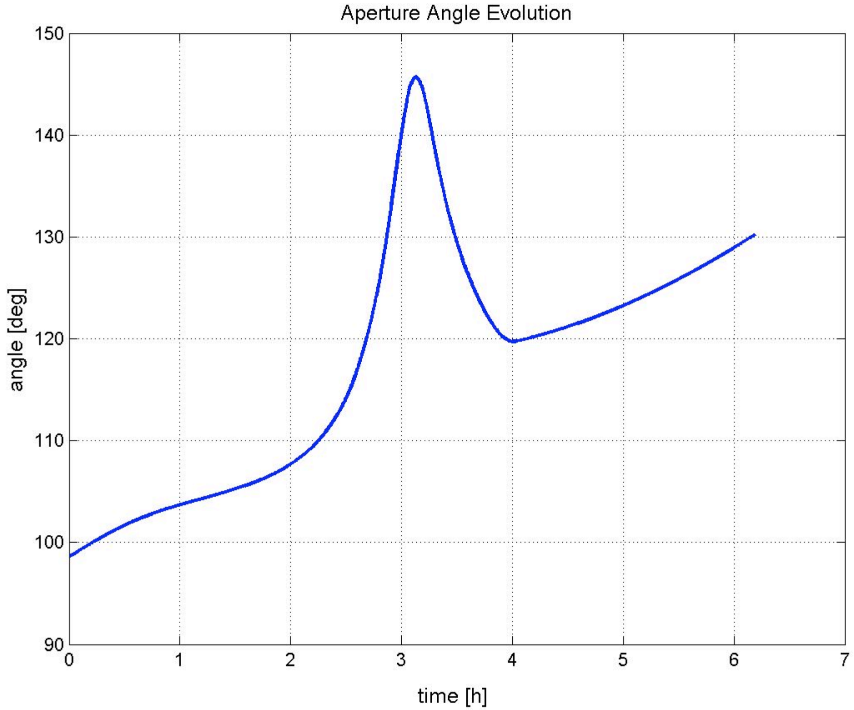


Projection in the *x*-*y* plan of the 3 spacecraft trajectories in LVLH

Guidance&Control → Matlab/Simulink simulator results

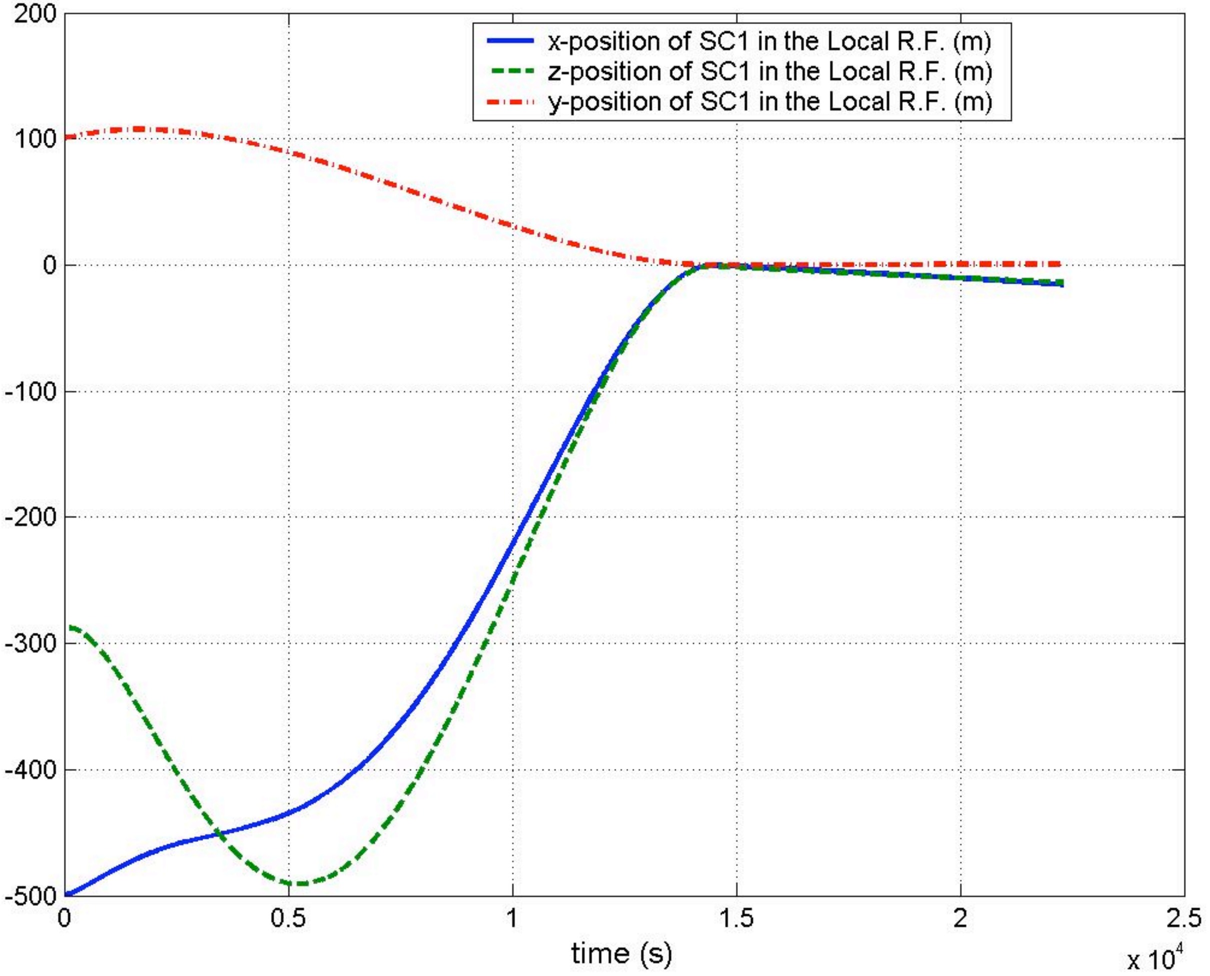


The evolution of the distances between the hub and TF1 (respectively TF2)



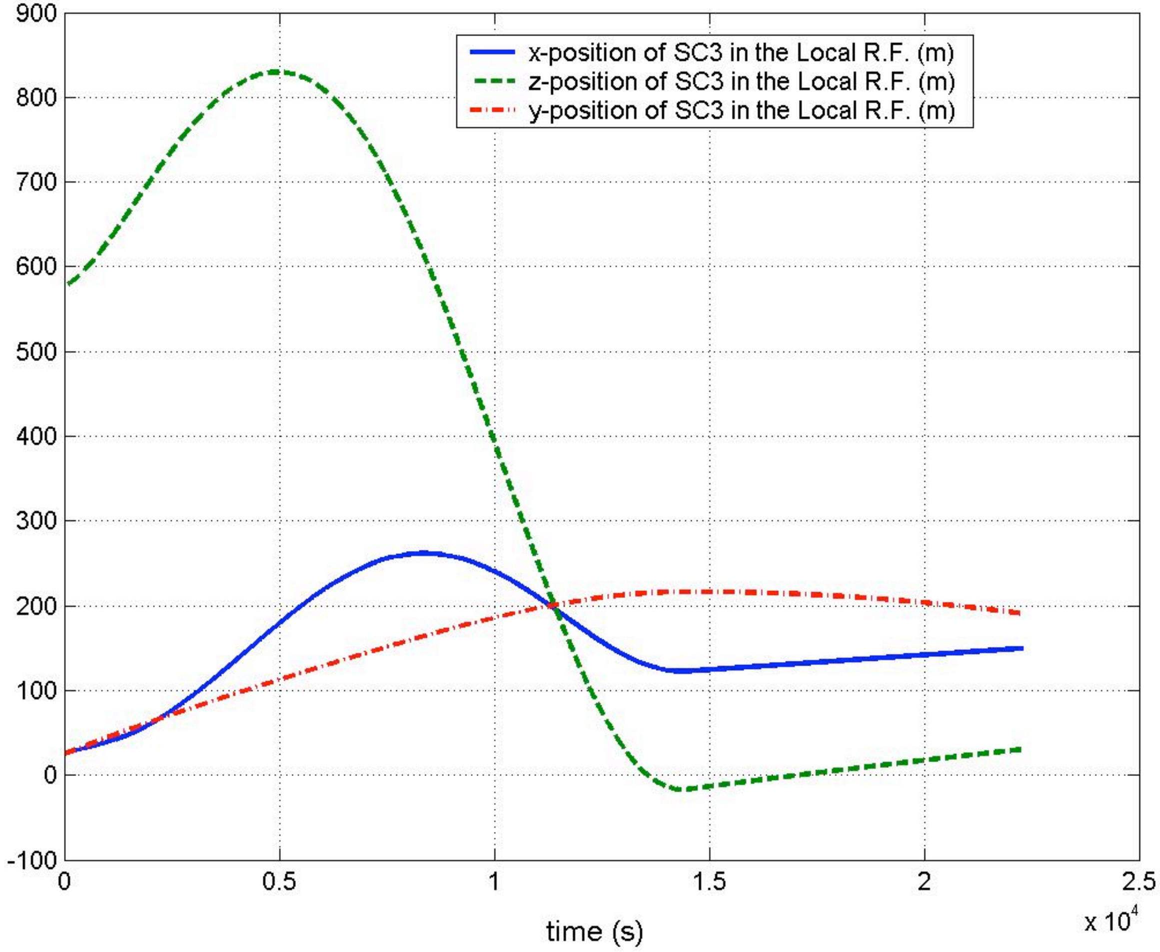
The evolution of the aperture angle of the triangle formation

Guidance&Control → Matlab/Simulink simulator results



Hub optimal trajectory positions (x_1, z_1 and y_1) with respect to time t

Guidance&Control → Matlab/Simulink simulator results



TF2 optimal trajectory positions (x_3, z_3 and y_3) with respect to time t

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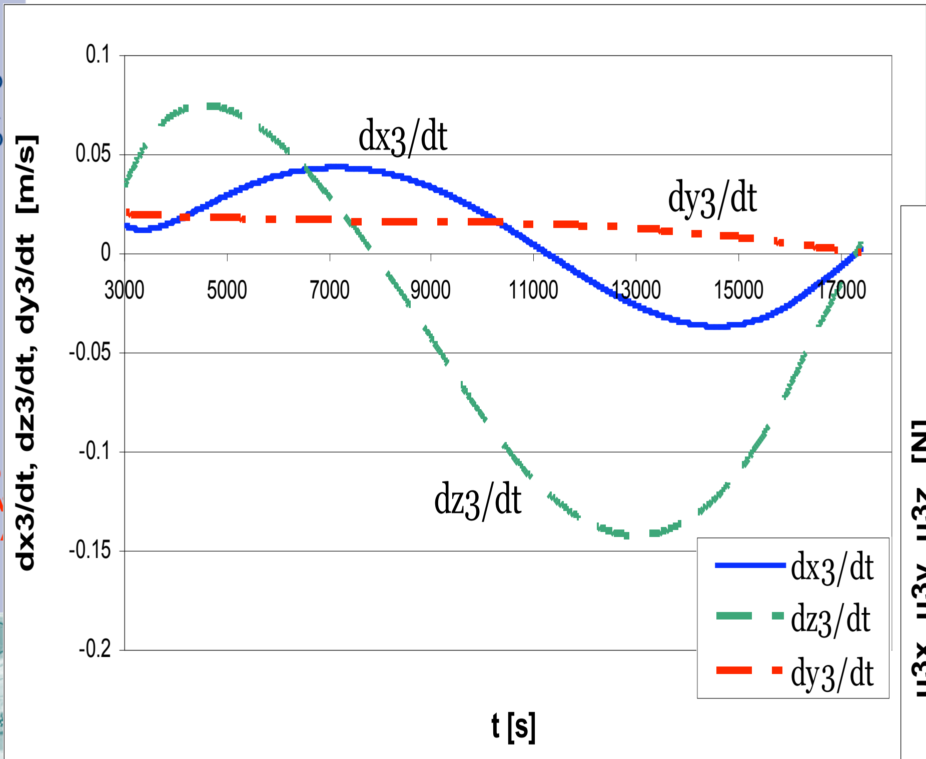
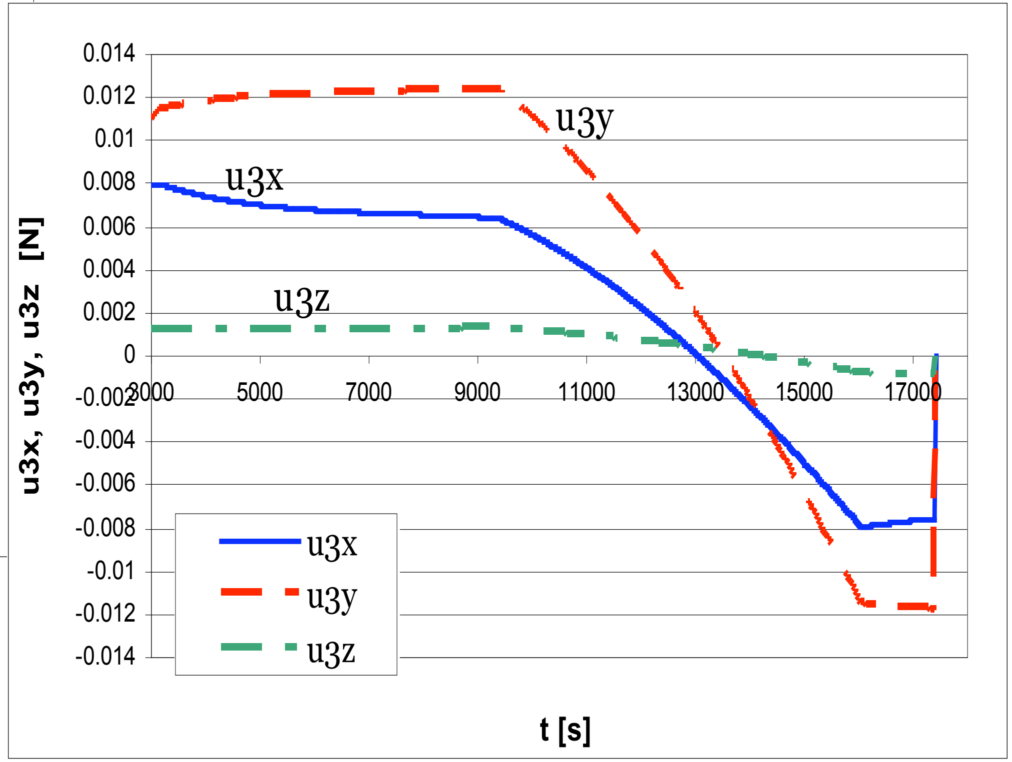


TF2 optimal trajectory velocities with respect to time *t*, in LVLH $(\dot{x}_3, \dot{z}_3 \; and \; \dot{y}_3)$



TF2 optimal control inputs ($u_{3,x}$, $u_{3,y}$ and $u_{3,z}$) with respect to time *t*, in IPQ

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Numerical conclusions

- By using the proposed PMP formulation, the optimal trajectory (positions and velocities) from the initial to the desired state is obtained, as well as the corresponding optimal control inputs.
- The error between the obtained state vector $\mathbf{X}(\theta_2)$ and the desired state vector $\mathbf{X}_{des}(\theta_2)$ is of the order of 1m for position components and of 10^{-3}m/s for velocity components.
- The computing time if of 5s, on a Pentium4 3.0GHz

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Conclusions

- Optimal trajectory planning algorithm for formation flying spacecraft, which allows the inclusion of actuators saturation and non-linear perturbation models in the dynamics equations
- We solve it by a Pontryagin Maximum Principle based iterative "shooting method"
- Thus, we obtain trajectories that require less control effort during the trajectory tracking phase of the mission; the spacecraft must not collide

Ø ***Important advance***: The closed loop LINEAR CONTROLLER based on the PMP formulation

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Following work

- Finish the implementation of the closed-loop linear controller in the simulator
- Realize also the ATTITUDE CONTROL
- Consider the different perturbations in the closed-loop linear controller, by finding the most appropriate linearized expressions of these perturbations
- Perform different tests, for example concerning the consideration of collision avoidance in our optimal control formulation