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Multi-Robot Localization with Partial Observations

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Topics

- Introduction
- Kalman Filter
- Sequential Update
- Partial Update
- Partial Update with Influence Relation
- Multi-Robot Localization
- Future Work

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Introduction

Problem: find the position (x, y, θ) of each mobile robot of a group of m robots using all information avaiable:

- Internal sensors that measure the self motion of the robot.
- External sensors that provide a representation of the environment.
- Position estimation comunicated by others robots.

Introduction

- Partial observation: an observation that provides partial information about the state.

- Assumption: even partial observations provide *some* information to improve the estimation of the state.

Kalman Filter

- Using Kalman filter to:
  - estimate robot position with associated measure of accuracy.
  - fuse informations from the different sensors.

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Kalman Filter

- System Model

$$x(k+1) = F(k)\,x(k) + w(k)$$

- Observation Model

$$z(k) = H(k)\,x(k) + v(k)$$

Kalman Filter Equations

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Prediction

$$\hat{x}(k+1|k) = F(k)\hat{x}(k|k)$$

$$P(k+1|k) = F(k)P(k|k)F^T(k) + Q(k)$$

$$\hat{z}(k+1|k) = H(k)\hat{x}(k+1|k)$$

Observation

$$v(k+1) = z(k+1) - \hat{z}(k+1|k)$$

$$S(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1)$$

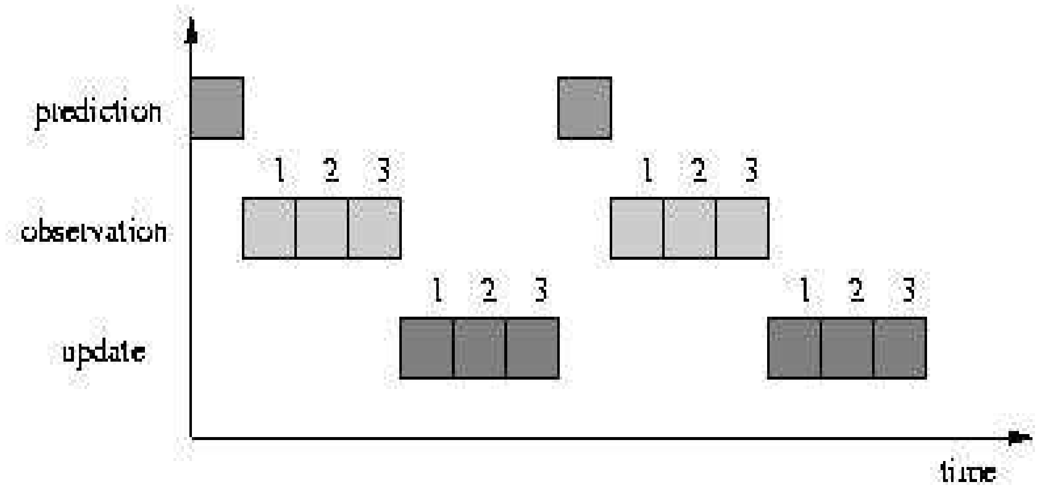
Update

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[z(k+1) - \hat{z}(k+1|k)]$$

$$P(k+1|k+1) = P(k+1|k) - K(k+1)H(k+1)P(k+1|k)$$

$$K(k+1) = P(k+1|k)H^T(k+1)S(k+1)^{-1}$$

Batch Update



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Sequential Update

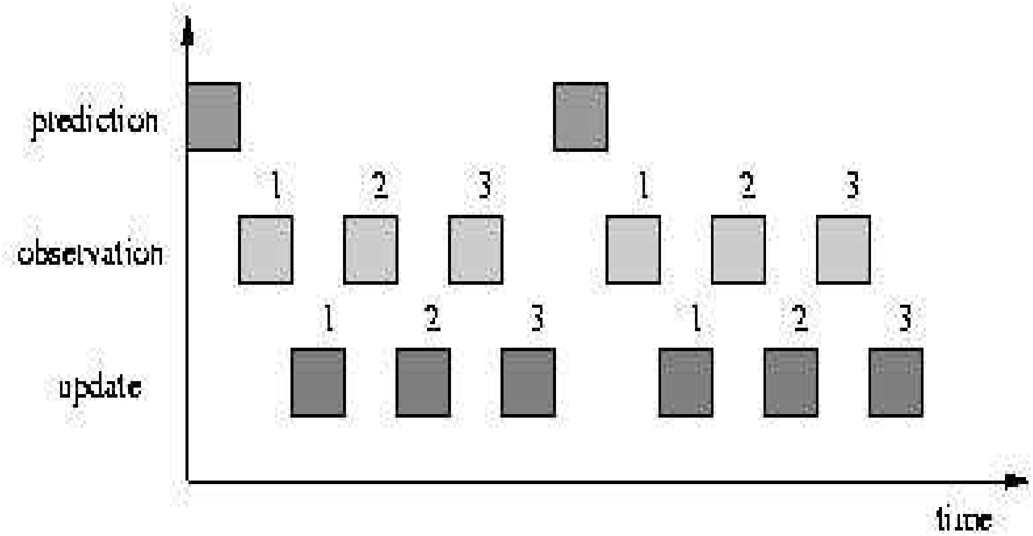
- Observation vector $z(k)$ can be partitioned into n subvectors with uncorrelated associated noise subvectors:

$$z(k) = [z_1(k), \ldots, z_n(k)]$$
$$H(k) = [H_1(k), \ldots, H_n(k)]$$

- State is updated sequentially for each observation subvector:

$$\hat{x}_i(k+1|k+1) = \hat{x}_i + K_i(k+1)[z_i(k+1) - \hat{z}_i(k+1|k)]$$
$$P_i(k+1) = P_i(k+1|k) - K_i(k+1)H_i(k+1)P_i(k+1|k)$$
$$K_i(k+1) = P_i(k+1|k) - K_i(k+1)H_i(k+1)S_i^{-1}(k+1)$$

Sequential Update



Observability

- Observation $z(k)$ is related to state $x(k)$ by observation matrix $H(k)$:

$$z(k) = H(k)\,x(k) + v(k)$$

- Observability defines the ability to determine the state $x(k)$ from the observation $z(k)$.

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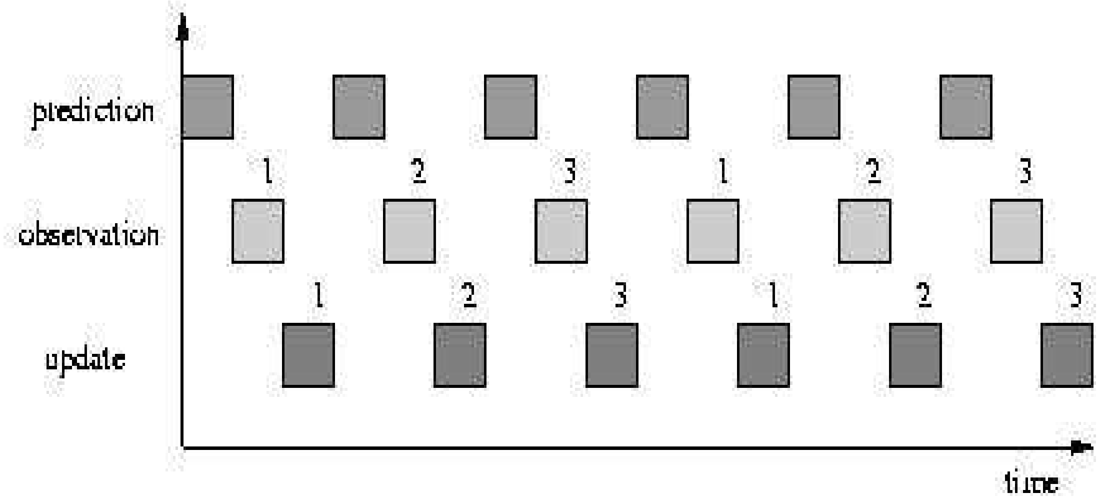
Partial Update

- A collection of locally incomplete observations results in a system that is globally observable.
- Incomplete observations can be used incrementally to improve an existing estimation.
- State is estimated by sequentially incorporating incomplete observations.

Partial Update

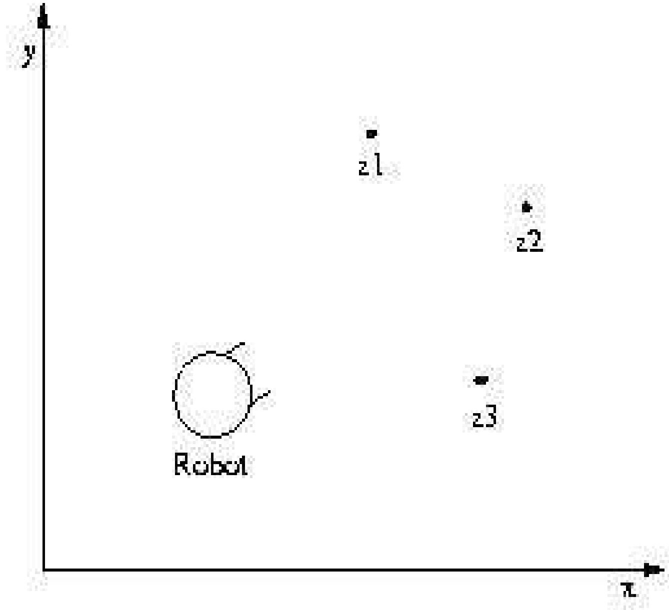


Partial Update with Influence Relation

- Assumptions:
  - an observation provides information to update only part of the state.
  - a priori information about the relation between observation and state.

Partial Update with Influence Relation: Example

- Observation z_1 is related with component x of state.
- Observation z_2 is related with component y of state.
- Observation z_3 is related with component θ of state.



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Partial Update with Influence Relation: Example

- Each observation has a vector of influence associated:

$$d_{z1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad d_{z2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad d_{z3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- Let the observation matrix:

$$H(k) = \begin{bmatrix} H_1(k) & H_2(k) & H_3(k) \end{bmatrix}$$

- For each observation a new observation matrix is formed. For the first observation:

$$H'_1(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} H(k) = \begin{bmatrix} H_1(k) & 0 & 0 \end{bmatrix}$$

Partial Update with Influence Relation

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- The observation matrix can be divided in two parts:

$$H(k) = \begin{bmatrix} H_z(k) & H_n(k) \end{bmatrix}$$
$$H'(k) = \begin{bmatrix} H_z(k) & 0 \end{bmatrix}$$

- The covariance matrix can be viewed as:

$$P = \begin{bmatrix} P_{zz} & P_{zn} \\ P_{nz} & P_{nn} \end{bmatrix}$$

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Partial Update with Influence Relation

- The innovation covariance matrix S and the Kalman gain K are function only of P_{zz} and H_z.

$$S = H_z P_{zz} H_{zz}^T + R$$

$$K = \begin{bmatrix} P_{zz} H_z^T S^{-1} \\ P_{nz} H_z^T S^{-1} \end{bmatrix}$$

- The covariance matrix is updated:

$$P = P - KHP$$

$$P = P - \begin{bmatrix} P_{zz} H_z^T S^{-1} H_z P_{zz} & P_{zz} H_z^T S^{-1} H_z P_{zn} \\ (P_{zz} H_z^T S^{-1} H_z P_{zn})^T & P_{nz} H_z^T S^{-1} H_z P_{zn} \end{bmatrix}$$

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Partial Update with Influence Relation: Constraints

- A priori information about which observations influence which part of the state.
- Observations are not correlated.
- Sufficient number of locally incomplete observations are collect such that the overall system is globally observable.

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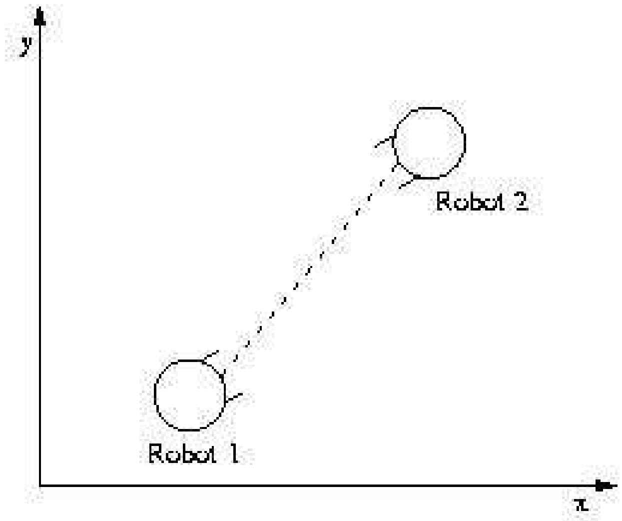
Multi-Robot Localization with Partial Observation

- Partial observations provided by:
  - sensors.
  - state estimation comunicated by the others robots.
- Comunicated estimation can be correlated.

Multi-Robot Localization with Partial Observation



Robot 1 observes robot 2 and robot 2 observes robot 1.

Multi-Robot Localization with Partial Observation

- Robot 1 observes robot 2. Observation of robot 2 is partial and can update only part of the state of robot 1.
- Robot 1 updates his state and comunicates his state to robot 2.
- Robot 2 update his state using his own observation of robot 1.
- Robot 2 fuses his state with the information provided by robot 1.

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Future Work

- State estimation of a group of robots using partial information:
  - Work on it...